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Enhanced Multi-Level Electro-Geometric Model for Assessing Lightning Performance of Transmission Lines Under Non-Vertical Strokes

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Abstract: In this paper, a procedure developed for evaluating lightning flashover rates caused by non-vertical strokes on transmission lines (TLs) is presented. To achieve this, a multi-level electro-geometric model (MEGM) is developed to determine the termination point of impact (TPI) of each lightning stroke on TLs in the presence of shield wires (SWs). The proposed method can detect strokes on each phase conductor and SWs separately. Then, a Monte Carlo simulation is performed using an electromagnetic transient program (ATP/EMTP) and MATLAB to estimate the lightning flashover rates. The outage rates are calculated for a test system through the proposed and conventional EGM models, and the results show that the lightning outage rate is not the same for different models. The difference in the estimated outage rate has a direct effect on the technical and economic protection strategy of power utilities. However, the results show that the outage rate calculated by the proposed EGM model is lower than that calculated by the conventional model and hence the proposed model predicts lower investment costs to implement the protection scheme for the power networks.

Keywords: **lightning flashover rate, Monte Carlo simulation, multi-level electro-geometric model (MEGM), transmission line protection**

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1. Introduction

Lightning strokes are essential when it comes to the insulation breakdown and outage rate of transmission lines (TLs). The assessment of the lightning performance of TLs is a key factor in the appropriate design of the protection scheme against lightning surges. For this purpose, a simple electro-geometric model (EGM) is conventionally used to determine the termination point of impact (TPI) of each lightning stroke on the TLs, assuming that the lightning strokes are vertical.

However, considering the actual behavior of the lightning strokes, this assumption is inaccurate. In other words, the modeling of non-vertical strokes is vital for the accurate prediction of lightning-related stresses and TL outages.

Lightning overvoltages have been analyzed in many studies using different methods, among which the EGM is the most widely used for calculating the lightning performance of the TLs.

In Anderson (2005), Datsios et al. (2023), the probabilistic behavior of lightning parameters is investigated, and the statistical distribution of parameters is discussed. In Anderson (1961), Nematollahi and Vahidi (2023), the statistical assessment of lightning overvoltages caused by vertical strokes is performed. First, the lightning parameters are generated based on the Monte Carlo method. Then, the resultant overvoltages are calculated by analytical relations that must be solved using numerical techniques whose accuracy might be an issue.

In Mousa and Srivastava (1989), Cao et al., (2024), assuming the location of lightning occurrence on the ground is known, a single-level EGM is used to determine the TPI on the transmission lines, i.e., stroke to ground, shield wires (SWs), or phase conductors. In these studies, the overlap of the maximum shielding failure current of which no strokes will terminate on the phase conductor, and the non-vertical strokes are not considered.

In General Electric Company (GE, 1982), Nicora et al., (2021), non-vertical strokes are discussed, and the statistical behavior of the stroke angle is obtained. Then, a single-level EGM is presented to determine the TPI on the transmission lines. Nevertheless, the presented approach can neither distinguish among strokes to each of the phase conductors and shield wires nor consider the displacement of the point of lightning occurrence on the ground.

Recent advances include the leader progression model (LPM) for higher accuracy (He et al., 2014; Datsios et al., 2023) and improved EGMs optimized with techniques like genetic algorithms (Hashemian et al., 2018). However, these either require intensive computation (LPM) or remain single-level in their geometric analysis. This work bridges the gap by proposing a multi-level EGM (MEGM) that maintains computational efficiency while providing conductor-specific impact assessment for non-vertical strokes.

The proposed multi-level EGM (MEGM) fundamentally advances the state of the art beyond the improved EGM in Hashemian et al. (2018) and the non-vertical stroke model in Shafaei and Shariatinasab (2012). While Hashemian et al. (2018) focuses on optimizing EGM parameters and Shafaei and Shariatinasab (2012) incorporates non-vertical angles into a single-level EGM, our MEGM introduces a novel multi-level geometric decision-making framework. This framework explicitly differentiates between strokes to each individual phase conductor and shield wire, a capability absent in both Hashemian et al. (2018) and Hashemian et al. (2018). Furthermore, MEGM systematically integrates the maximum shielding failure current and the stroke location to activate one of five distinct analytical modes, providing a more granular and accurate prediction of the termination point of impact for non-vertical strokes.

This study presents an improved EGM developed to determine the TPI on transmission lines for non-vertical strokes. The proposed multi-level EGM (MEGM) estimates the TPI accurately and can distinguish between strokes hitting to each one of the phases and shield conductors, regardless of the structure characteristics, point of lightning occurrence on the ground, and stroke angle. A sample TL is then implemented in ATP/EMTP linked to MATLAB to obtain the overvoltage and outage rates caused by non-vertical strokes. Finally, the calculated results are compared with those of previous studies, and a discussion is presented. A comparative analysis of the key features of different lightning assessment approaches is provided in Table 1, highlighting the distinct advantages of the proposed MEGM.

2. Determining TPI Using Conventional EGM

To determine the TPI on a transmission line for non-vertical strokes, the so-called impact area must be defined. Then, the stroke angle is obtained from the angle between the lightning leader channel and the vertical line, where the lightning strikes the ground. Finally, the termination point of impact is determined by the EGM.

Table 1. Comparative analysis of lightning performance assessment methods.

Feature / Method	Conventional EGM	Improved EGM (Hashemian et al., 2018)	LPM (He et al., 2014; Datsios et al., 2023)	Proposed MEGM
Handles non-vertical strokes	No	Limited	Limited	Yes
Discriminates strokes to individual phase conductors	No	No	No	Yes
Computational Efficiency	High	High	Low	High
Multi-level geometric decision logic	No	No	No	No
Considers max. shielding failure current	No	Yes	Yes	Yes
Prediction accuracy vs. field data	Low	Moderate	Moderate	High
Primary Application	Basic screening	Parameter-optimized design	Detailed physical analysis	Accurate engineering assessment

2.1 Impact Area

The lightning strokes terminating on the ground are located within an impact area (Figure 1). The (x, y) coordinates of the lightning stroke on the ground must be acquired as uniformly distributed over the impact area (Mousa & Srivastava, 1989; Shariatinasab et al., 2009a; Martinez & Castro-Aranda, 2006). However, the length of the impact area is equal to the length of the span, and the width (d) is derived from the application of the EGM, corresponding to the maximum peak current magnitude (Martinez & Castro-Aranda, 2006).

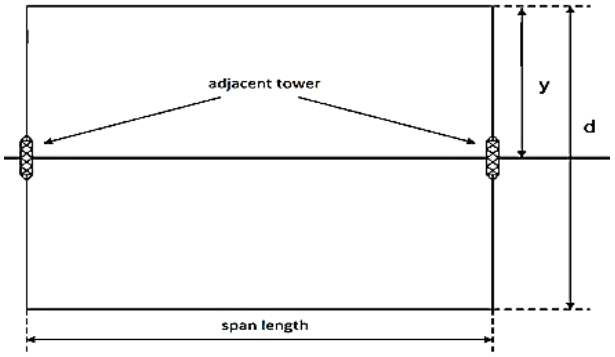


Figure 1. The impact area.

2.2 Distribution of Stroke Angle

The stochastic behavior of the stroke angle is usually displayed with a probability density function, as follows (Hileman, 2018):

$$p(\Psi) = \begin{cases} 0 & \Psi < -\frac{\pi}{2} \\ k\cos^m\Psi & -\frac{\pi}{2} < \Psi < \frac{\pi}{2} \\ 0 & \Psi > \frac{\pi}{2} \end{cases} \quad (1)$$

where Ψ is the angle of the lightning leader channel from the vertical direction, and m is a constant. For a given m, the value of the coefficient k can be evaluated as follows (Martinez & Castro-Aranda, 2006):

$$k = \left(\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^m\Psi d\Psi \right)^{-1} \quad (2)$$

The type of distribution function of the stroke angle is a function of m. For example, for $m=0$, the angle distribution function will be uniform, while for values of m≤2, the density function tends to be a Gaussian distribution function (Martinez & Castro-Aranda, 2006). Usually, the value of $m = 2$ is used in the calculations (Shafaei & Shariatinasab, 2012).

If the impact point of the lightning on the ground, P_1, is considered as the origin, as shown in Figure 2, the stroke angle relative to the vertical line can be distinguished by its positive or negative value. The lightning leader channel with a positive stroke angle terminates on the ground on the right side of the vertical line, and the stroke with a negative value terminates on the ground on the left side.

It can be concluded that a stroke located at P_1 (Figure 2a) would not impact the wire if the leader channel were vertical or the stroke angle were negative. However, if the ground location of the lightning is just below the wire (Figure 2b), the vertical and non-vertical strokes with stroke angles of ($\Psi_1 \leq \Psi \leq \Psi_2$) will reach the wire.

2.3 Determining TPI for Non-Vertical Strokes

The flashover can be caused by a stroke to shield wires (SWs) or phase conductors. Given the location (*x,y*) of the lightning occurrence in the impact area, the termination point of impact on TLs can be determined from the EGM, as shown in Figure 3.

The striking distances of a lightning stroke to the conductors and the ground are calculated as follows (Telló, 2007):

$$R_c = 10I^{0.65} \tag{3}$$

$$\begin{cases} \mathrm{R_g} = \beta \mathrm{R_c} \\ \beta = 0.36 + 0.17\ln(43 - \mathrm{H})\mathrm{H} < 40\mathrm{m} \\ \beta = 0.55\mathrm{H} \geq 40\mathrm{m} \end{cases} \tag{4}$$

where R_c and R_g are, respectively, the striking distances to the conductor (phase or SW) and the ground, and H is the mean height of the phase conductor:

$$\mathrm{H} = \mathrm{H_{ph}} - \tfrac{2}{3}\mathrm{H_{ms}} \tag{5}$$

where H_{ph} and H_{ms} are the phase conductor heights at the tower and midspan, respectively.

From Figure 3, the unshielded area D_c and the striking distance associated with the shield wire D_g are given by:

$$\begin{cases} \mathrm{D_c} = \mathrm{R_c}.(\cos\theta - \cos(\alpha + \beta)) \\ \mathrm{D_g} = \mathrm{R_c}.(\cos(\alpha - \beta)) \end{cases} \tag{6}$$

where

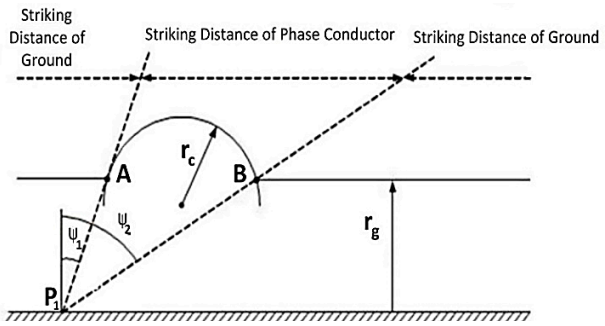
$$\begin{cases} \alpha = \tan^{-1}\left(\frac{\mathrm{d}}{\mathrm{H_g}} - \mathrm{H_{ph}}\right) \\ \theta = \sin^{-1}\left(\mathrm{R_g} - \frac{\mathrm{H_{ph}}}{\mathrm{R_c}}\right) \\ \beta = \sin^{-1}\left((\mathrm{H_g} - \mathrm{H_{ph}}).\frac{\sqrt{1+\tan^2\alpha}}{2\mathrm{R_c}}\right) \end{cases} \tag{7}$$

The height of the point of intersection between the striking distances of the outer phase conductor and the outer shield wires is also calculated by:

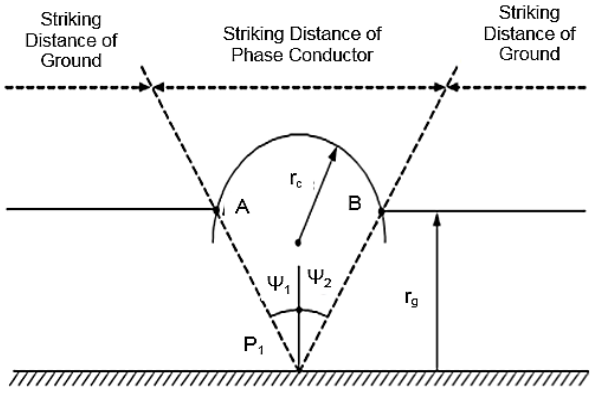
$$L = H_{ph} + R_c.\sin(\alpha+\beta) \tag{8}$$

The maximum shielding failure current I_{max}, beyond which all the lightning currents strike the shield wire, is defined as follows (Hileman, 2018):

$$\mathrm{I_{max}} = \left(\frac{\mathrm{R_{gm}}}{\mathrm{A}}\right)^{\frac{1}{\mathrm{b}}} \tag{9}$$



a. Ψ_1 and Ψ_2 are positive.



b. Ψ_1 is negative, and Ψ_2 is positive.

Figure 2. Geometry for the stroke angle of non-vertical strokes.

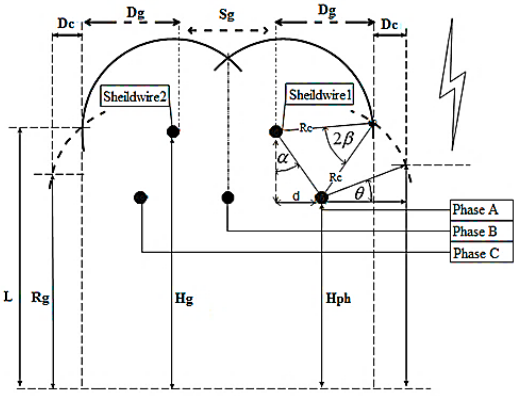


Figure 3. The conventional EGM model.

where

$$\begin{aligned} \mathrm{R_{gm}} &= \left(\frac{\mathrm{H_g}+\mathrm{H_{ph}}}{2\mathrm{K_0}}\right) \times \mathrm{K_1} \\ \mathrm{K_1} &= 1 + \sqrt{1 - \mathrm{K_0}\left(1 + \left(\frac{\mathrm{D_c}}{\mathrm{H_g}+\mathrm{H_{ph}}}\right)^2\right)} \\ \mathrm{K_0} &= 1 - \left(\left(\frac{\mathrm{R_c}}{\mathrm{R_g}}\right).\sin^2\alpha\right) \end{aligned} \tag{10}$$

It is acknowledged that the estimation of the maximum shielding failure current (I_{max}) relies on established analytical formulas (Hileman, 2018; Telló, 2007), which is a common practice in EGM-based studies due to the challenges in obtaining directly measured data for this parameter across diverse network configurations. While this approach provides a robust and practical framework, future work could focus on correlating these analytical results with field-measured data from specific high-voltage line monitoring systems to further refine accuracy.

3. The Proposed MEGM Method

Considering striking distances and the maximum shielding failure current, a precise multi-level EGM can be established to determine the TPI for non-vertical strokes on the TLs in the presence of shield wires. The proposed EGM is categorized into two levels based on the lightning peak current, maximum shielding failure current, and the location of lightning occurrence on the ground surface.

It is important to note that the MEGM uses standard equations for basic lightning parameters (angle, striking distance, shielding current). The new contribution of this paper is the multi-level decision-making system (the 5 modes in Figures 4-8 and Eqs. 11-23) that determines exactly which wire the lightning will hit.

The mathematical formulation of the MEGM is organized into five distinct sets of equations, each corresponding to a specific operational mode. This structural separation is essential as each mode addresses unique geometric configurations and boundary conditions for lightning termination. While the equations may appear similar at a glance, they contain critical differences in parameters and application ranges that are fundamental to the model's accuracy in predicting the termination point of impact.

3.1 Distribution of Stroke Angle

The first level is used to specify the TPI of a lightning stroke in cases in which the peak current magnitude is less than the maximum shielding failure current. At this level, the lightning stroke may hit the phase conductor, shield wires, or the ground. Therefore, depending on the location of lightning occurrence on the ground surface, three modes can be established to specify the TPI on the transmission line.

First Mode

In this mode, as shown in Figure 4, the *y-coordinate* of the stroke point is located under the protective shadow of the SW, which means that $|y|<((S_g/2)+D_g)$.

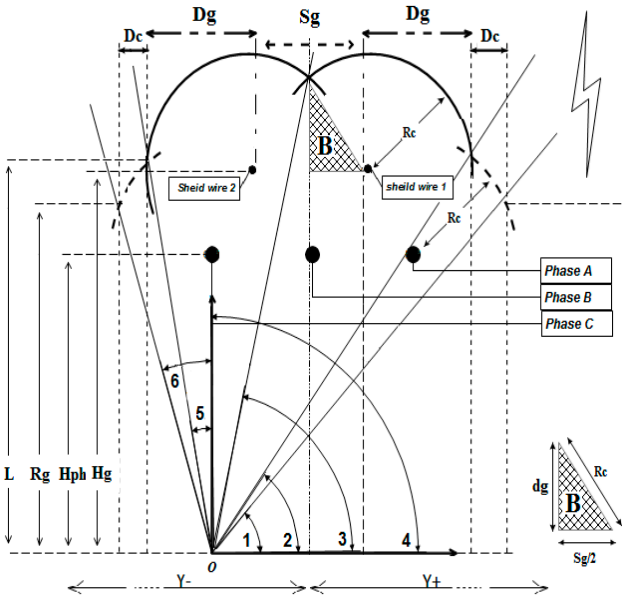


Figure 4. MEGM: First level-first mode (strokes under shield wire zone).

Supposing that the vertical lightning stroke strikes the ground surface at point *O* (Figure 4), the termination point of impact would be a shield wire 2. In the case of non-vertical lightning strokes, depending on the stroke angle terminated to the ground surface, the TPI would be different. To distinguish all possible states, six angles φ_1 to φ_6 must be determined, whose numerical indices are specified as 1 to 6 without the prefix φ in Figure 4.

For example, if the impact angle Ψ of the lightning stroke that hits point O lies between φ1 and φ2 or φ5 and φ6, the lightning surge strokes to phase A or C, respectively. If Ψ falls between φ2 and φ3, the lightning hits shield wire 1, while for the value of Ψ between (φ3-φ4) and φ5, the lightning stroke is expected to hit shield wire 2.

To specify the TPI for non-vertical strokes, the sides of the right triangle *B* must be calculated, in which the horizontal side is equal to half the distance between SWs, and its hypotenuse is equal to the radius of striking distance (R_c). However, the vertical side (d_g) is given by:

$$\mathrm{D_g} = \left(\mathrm{R_c^2} - \left(\frac{\mathrm{S_g}}{2}\right)^2\right)^{-\frac{1}{2}} \tag{11}$$

The angles φ_1 to φ_6 can be calculated as follows:

$$\begin{cases} \varphi_1 = \tan^{-1}\left(\mathrm{R_g}/(\mathrm{y} + \mathrm{S_g}/2 + \mathrm{D_g} + \mathrm{D_c})\right) \\ \varphi_2 = \tan^{-1}(\mathrm{L}/(\mathrm{y}+\mathrm{S_g}/2+\mathrm{D_g}+\mathrm{D_c})) \\ \varphi_3 = \tan^{-1}((\mathrm{H_g}+\mathrm{d_g})/\mathrm{y}), \varphi_4 = 90 \\ \varphi_5 = -\tan^{-1}((\mathrm{S_g}/2+\mathrm{D_g})-\mathrm{y}/\mathrm{L}) \\ \varphi_6 = -\tan^{-1}((\mathrm{S_g}/2+\mathrm{D_g}+\mathrm{D_c})-\mathrm{y}/\mathrm{R_g}) \end{cases} \tag{12}$$

Given the angles φ_1 to φ_6, the TPI on the transmission line for non-vertical lightning with an impact angle of Ψ is determined by:

$$\begin{cases} \text{if}\Psi > (\varphi_4 - \varphi_1)\text{or}\Psi \le \varphi_6\text{stroketoground} \\ \quad \text{if}\Psi \le (\varphi_4-\varphi_1)\&\Psi > (\varphi_4-\varphi_2)\text{stroketophaseA} \\ \quad \text{if}\Psi \le (\varphi_4-\varphi_2)\&\Psi > (\varphi_4-\varphi_3)\text{stroketoSW1} \\ \quad \text{if}\Psi \le (\varphi_4-\varphi_3)\&\Psi > \varphi_5\text{stroketoSW2} \\ \quad \text{if}\Psi \le \varphi_5\&\Psi > \varphi_6\text{stroketophaseC} \end{cases} \quad (13)$$

Based on Equation (13), the analysis demonstrates that the shielding failure related to phases A and C can be distinguished by the presented MEGM model.

Second Mode

In this mode, it is assumed that the y-coordinate of the stroke point terminating on the ground surface is located within the protective shadow of the shield wires, and the peak current magnitude is less than the maximum shielding failure current (Figure 5). In this case, it can be seen that $(((S_g/2)+D_g+D_c) > |y| \ge ((S_g/2)+D_g))$.

To specify the impact point, the unknowns of triangle *B* and then the angles φ_1 to φ_6 must be determined. In this circumstance, the values of φ_1, φ_2, and φ_3 can be calculated as described previously, and the remaining angles are determined as follows:

$$\begin{cases} \varphi_4 = \tan^{-1}(\mathrm{L}/(\mathrm{y} - \mathrm{S_g}/2 - \mathrm{D_g})), \varphi_5 = 90^{\circ} \\ \varphi_6 = -\tan^{-1}\left((\mathrm{S_g}/2 + \mathrm{D_g} + \mathrm{D_c} - \mathrm{y})/\mathrm{R_g}\right) \end{cases} \quad (14)$$

Similar to the first mode, the TPI of non-vertical strokes with impact angle Ψ that terminate on the ground surface at point *O* can be determined by comparing Ψ with φ_1 to φ_6:

$$\begin{cases} \text{if}\Psi > (\varphi_5 - \varphi_1)\text{or}\Psi \le \varphi_6\text{stroketoground} \\ \quad \text{if}\Psi \le (\varphi_5-\varphi_1)\&\Psi > (\varphi_5-\varphi_2)\text{stroketophaseA} \\ \quad \text{if}\Psi \le (\varphi_5-\varphi_2)\&\Psi > (\varphi_5-\varphi_3)\text{stroketoSW1} \\ \quad \text{if}\Psi \le (\varphi_5-\varphi_3)\&\Psi > (\varphi_5-\varphi_4)\text{stroketoSW2} \\ \quad \text{if}\Psi \le (\varphi_5-\varphi_4)\&\Psi > \varphi_6\text{stroketophaseC} \end{cases} \quad (15)$$

For example, if the stroke angle Ψ is between φ_1 and φ_2 or (φ_4 - φ_5) and φ_6, the lightning strokes are to phase A or C, respectively, which means that the shielding failures related to phases *A* and *C* can be distinguished.

Third Mode

As shown in Figure 6, if the y-coordinate of the stroke on the ground surface is located outside the protective shadow of the shield wires and the striking distance of the phase conductor, the vertical strokes inevitably terminate on the ground. However, there is a possibility of TPI on the shield wire or phase conductor for non-vertical strokes.

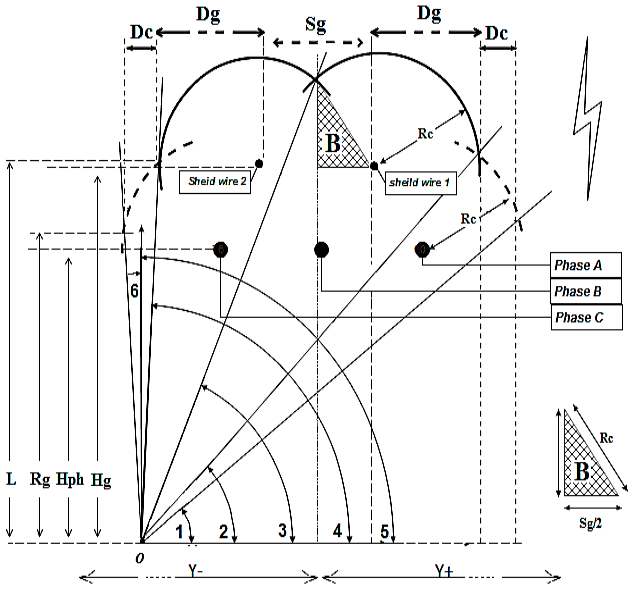


Figure 5. MEGM: First level-second mode (strokes at protection zone edge).

In this case, angles φ_1, φ_2, φ_3, and φ_4 are calculated similarly to the second mode; while φ_5 and φ_6 are given by:

$$\begin{cases} \varphi_5 = \tan^{-1}(\mathrm{R_g}/(\mathrm{y} - \mathrm{S_g}/2 - \mathrm{D_g} - \mathrm{D_c})) \\ \varphi_6 = 90^{\circ} \end{cases} \quad (16)$$

Now, for the non-vertical stroke of the angle of Ψ, the TPI is determined as follows:

$$\begin{cases} \text{if}\Psi > (\varphi_6 - \varphi_1)\text{or}\Psi \le (\varphi_6 - \varphi_5)\text{stroketoground} \\ \quad \text{if}\Psi \le (\varphi_6-\varphi_1)\&\Psi > (\varphi_6-\varphi_2)\text{stroketophaseA} \\ \quad \text{if}\Psi \le (\varphi_6-\varphi_2)\&\Psi > (\varphi_6-\varphi_3)\text{stroketoSW1} \\ \quad \text{if}\Psi \le (\varphi_6-\varphi_3)\&\Psi > (\varphi_6-\varphi_4)\text{stroketoSW2} \\ \quad \text{if}\Psi \le (\varphi_6-\varphi_4)\&\Psi > (\varphi_6-\varphi_5)\text{stroketophaseC} \end{cases} \quad (17)$$

3.2 The Second Level of MEGM

This level is used to determine the TPI for the strokes in which the peak current magnitude is higher than the maximum shielding failure current. Under this condition, the striking distance of shield wires completely covers the striking distance of the outer phase conductor, i.e., $D_c = 0$. Therefore, the strike of the lightning surge to the phase conductor is not possible. However, depending on *the y-coordinate* of the lightning stroke, the following two cases are predictable.

First Mode

In this mode, $|y| < ((S_g/2) + D_g')$ and the unknowns of the two triangles, *A* and *B*, must be determined before calculating the desired angles (Figure 7). The unknowns of triangle *B* can be obtained as described before. However, to determine the sides of triangle *A*, the parameter δ must be calculated first:

$$\begin{cases} \delta = 0 \text{if} R_g \le H_g \\ \delta = \sin^{-1}\left((R_g - H_g)/R_c\right) \text{if} R_g > H_g \end{cases} \quad (18)$$

where $dx\ (=D_c')$ is,

$$dx = \frac{R_c}{\cos\delta} \quad (19)$$

The hypotenuse of triangle *A* is equal to the striking distance radius. Therefore, the vertical side is defined using the difference between the shield wire height and the ground striking distance. It should be mentioned that if $\delta=0$, then $R_g=H_g$. Based on the above description, the angles φ_1, φ_2, φ_3, and φ_4 are given by:

$$\begin{cases} \varphi_1 = \tan^{-1}\left(R_g/(y + S_g/2 + dx)\right) \\ \varphi_2 = \tan^{-1}((H_g + d_g)/y), \varphi_3 = 90^\circ \\ \varphi_4 = -\tan^{-1}\left((S_g/2 + dx - y)/R_g\right) \end{cases} \quad (20)$$

Then, the TPI on the shield wire or the ground surface can be determined for the non-vertical stroke with impact angle Ψ by the following equation:

$$\begin{cases} \text{if}\Psi > (\varphi_3 - \varphi_1) \text{or} \Psi \le \varphi_4 \text{stroketoground} \\ \text{if}\Psi \le (\varphi_3 - \varphi_1) \text{and} \Psi > (\varphi_3 - \varphi_2) \text{stroketoSW1} \\ \text{if}\Psi \le (\varphi_3 - \varphi_2) \text{and} \Psi > \varphi_4 \text{stroketoSW2} \end{cases} \quad (21)$$

Second Mode

As shown in Figure 8, in this mode, the y-coordinate of a striking point on the ground is located outside the protective shadow of the shield wires. In this mode, the angles φ_1 and φ_2 and the unknowns of triangles *A* and *B* are calculated similarly to the first mode, while φ_3 and φ_4 are given by:

$$\varphi_3 = \tan^{-1}(R_g/(y - S_g/2 - dx)), \varphi_4 = 90° \quad (22)$$

Finally, for the non-vertical stroke of angle Ψ, the TPI is determined by:

$$\begin{cases} \text{if}\Psi > (\varphi_4 - \varphi_1) \text{or} \Psi \le (\varphi_4 - \varphi_3) \text{stroketoground} \\ \text{if}\Psi \le (\varphi_4 - \varphi_1) \text{and} \Psi > (\varphi_4 - \varphi_2) \text{stroketoSW1} \\ \text{if}\Psi \le (\varphi_4 - \varphi_2) \text{and} \Psi > (\varphi_4 - \varphi_3) \text{stroketoSW2} \end{cases} \quad (23)$$

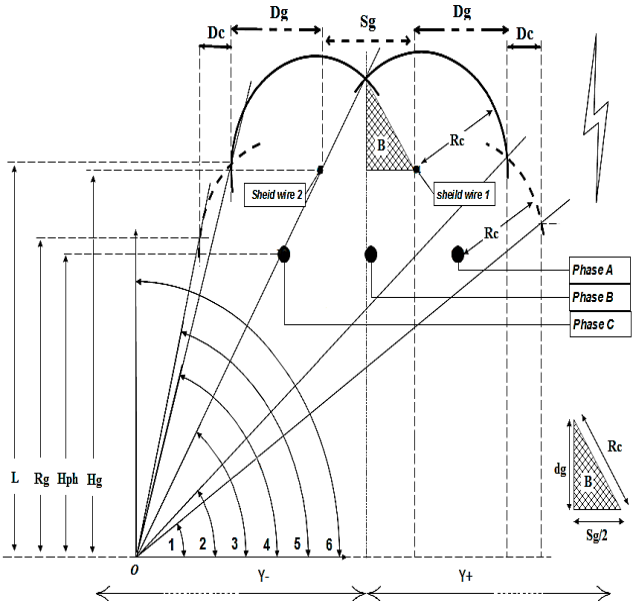


Figure 6. MEGM: First level-third mode (strokes outside protection zone).

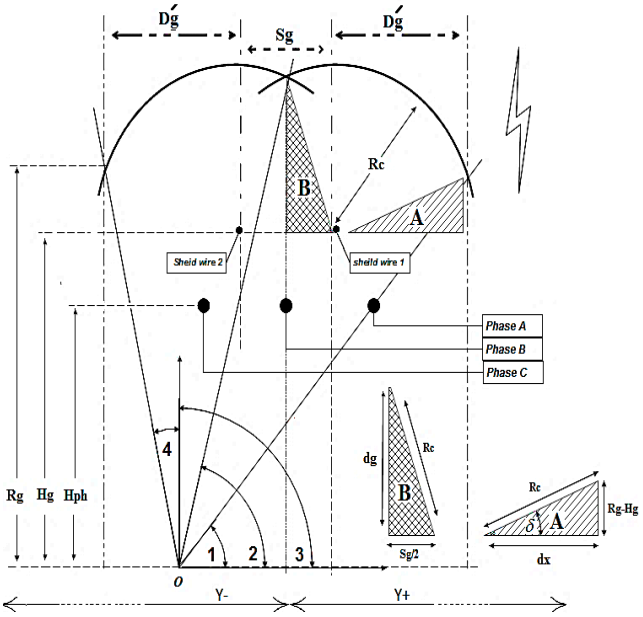


Figure 7. MEGM: Second level first mode (high-current strokes under shield wire).

In general, the presented MEGM is divided into two levels and five modes that can be categorized depending on the lightning current magnitude and the coordinates of the lightning point of impact on the ground. Figure 9 shows an instructive flowchart for using the appropriate mode/level of the MEGM.

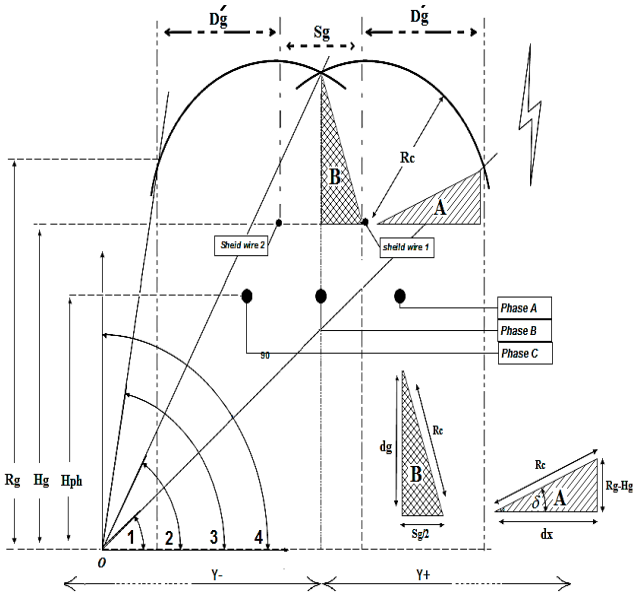


Figure 8. MEGM: Second level-second mode (high-current strokes outside shield wire).

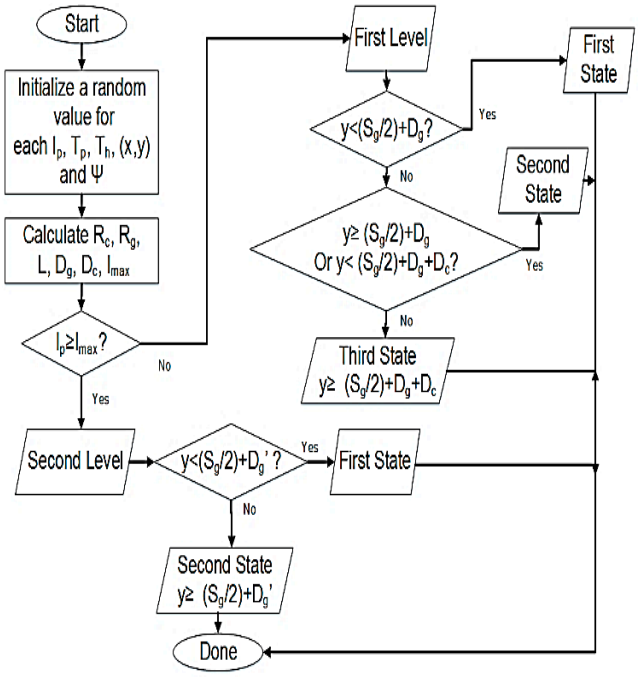


Figure 9. Flowchart of the proposed EGM (MEGM decision flowchart).

4. Monte Carlo Simulation

The Monte Carlo method is employed in ATP/EMTP simulations using MATLAB software, and the outage rate related to non-vertical lightning strokes is calculated. The Monte Carlo procedure for this purpose is defined as follows:

1. Generate the random value of the lightning waveform (based on the log-normal distribution function), the striking point coordinates on the ground, the lightning striking angle (based on the probability density function of Equation 1), the phase angle of power frequency voltage (based on a uniform distribution function between 0 and 360), the length of the insulator string, and the footing resistance (for more details, see Refs. (Shariatinasab et al., 2009a; General Electric Company [GE], 1982 ;Martinez & Castro-Aranda, 2006).
2. Select the appropriate level of the MEGM depending on the lightning current and coordinates of the striking point on the ground surface, and then determine the TPI on the TL (phase conductor, SW, or ground). Once the termination point of impact on the transmission line has been established, the corresponding lightning waveform is applied to the transmission line in ATP/EMTP, and the lightning overvoltages are calculated.
3. In each iteration, if the resultant overvoltage across the insulator string is higher than the insulator string strength, the number of outages due to lightning is increased by one.

4.1. Simulation Robustness and Convergence

To ensure the statistical reliability of the Monte Carlo simulation, a convergence analysis was performed. The simulation was run with an increasing number of iterations (N). The annual lightning flashover rate (LFOR) was monitored, and it was observed that the results stabilized with negligible variation (±0.5%) beyond N = 40,000 simulations. This sample size (N = 40,000) was therefore selected for all presented results to guarantee convergence. Furthermore, sensitivity tests were conducted on key parameters, including the soil resistivity (ρ) and the footing resistance (R_o). The results confirmed that the relative superiority of the proposed MEGM over the conventional EGM methods remained consistent across the tested range of parameter variations, underscoring the robustness of the main findings.

5. Test System

5.1 Modeling Guidelines

The point of impact of a lightning stroke can be determined using the following procedure: first, the ground location is randomly determined using a uniform ground distribution (as with vertical leader channels); next, the stroke angle is randomly determined; finally, the MEGM model is applied to distinguish the termination point of impact on the TL. The proposed method is used for a test

system that is a 400 kV transmission line in Iran's power network (Shariatinasab et al., 2009b). The single-line diagram of the transmission line is shown in Figure 10.

The design details are given as follows:

- The JMarti model is used to model the TL up to five spans away from the impact point. The span length is 450 m.
- The multi-layer model is used for the representation of the towers, whose parameters are entirely described in (Martinez & Castro-Aranda, 2005).
- The footing resistance is modeled by the following nonlinear resistance model:

	Conductor Type	Radius (cm)	Resistance (Ω/km)
Phase conductors	CARDINAL	3.04	0.061
Shield wire	7N8 AW	0.98	1.49

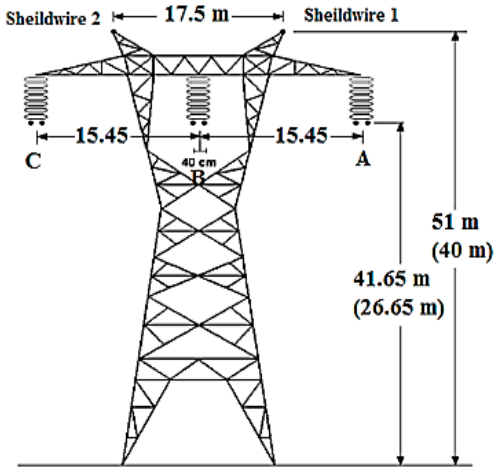


Figure 10. Typical line configuration (values in brackets are midspan heights).

where I is the footing resistance current magnitude, R_0 is the footing resistance, E_0 (=400 kV/m) is the soil ionization gradient, and ρ is the soil resistivity (1000 Ωm). To generate different random values of footing resistance, a normal distribution with mean and standard deviation values equal to 148 Ω and 5 Ω, respectively, is used (Shariatinasab et al., 2009a).

- The insulator string is modeled based on the leader progression model. In this model, depending on the momentary lightning overvoltage, the variation of the distance of the unbridged gap can be calculated. If the distance of the unbridged gap is longer than the distance of the insulator string, the flashover occurs (Filipović-Grčić et al., 2012):

$$\frac{dl}{dt} = k_1 \times U\left(t\right) \times \left(\frac{U(t)}{g-l} - E_{10}\right) \quad (24)$$

where $U(t)$ is the voltage across the insulator string in kV, g is the air gap in meters, l is the distance of the unbridged gap in meters, E_{10} $(=570\ kV/m)$ is the gradient at which the breakdown process starts, and k_1 $(=13\times10^{-7})$ is a constant (Shariatinasab et al., 2009a).

5.2 Simulation Results

For statistical evaluation of lightning surges, first, the main parameters are generated according to the standard values of lightning parameters (see Ref. (Shariatinasab et al., 2009a) for more details). Given the lightning waveform parameters, the coordinates (x, y) of each lightning stroke on the impact area (Figure 2) are generated. The maximum value of x is the span length, and the maximum value of y is determined corresponding to the maximum lightning current magnitude that is assumed to be 244 kA. The impact angle is generated from Eq. (1) for $m=2$ and $k=\pi/2$. Figure 11 shows the frequency distribution of 40,000 generated angles. Finally, the TPI of each stroke is determined using the presented MEGM. Then, the associated lightning stroke is connected to the termination point of impact (shield wire, phase conductor, or ground) as a current source in ATP/EMTP software.

Figures 12 to 14 show the frequency distribution of lightning peak current on the phase conductor, shield wire, and the ground.

For comparison purposes, the lightning-related outage rates are calculated by three methods as follows:

1) Method 1: It is assumed that the lightning stroke is non-vertical and the TPI is determined based on the proposed MEGM model.

2) Method 2: It is assumed that the lightning stroke is non-vertical and the conventional EGM is used (Shafaei & Shariatinasab, 2012).

3) Method 3: It is assumed that the lightning stroke is vertical and the conventional EGM is used (Shariatinasab et al., 2009a).

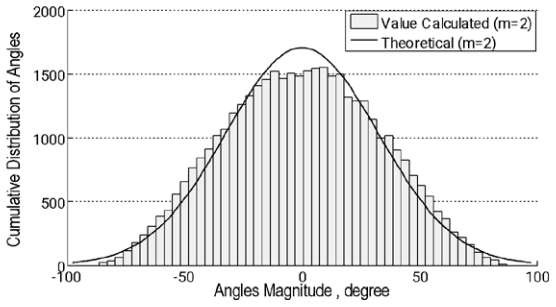


Figure 11. Frequency distribution of the generated angles (k=2/π, m=2)

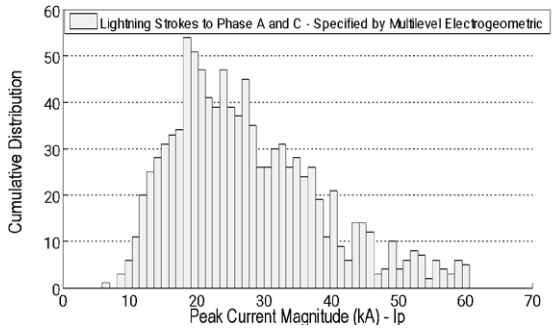


Figure 12. Frequency distribution of lightning peak current on phases A and C.

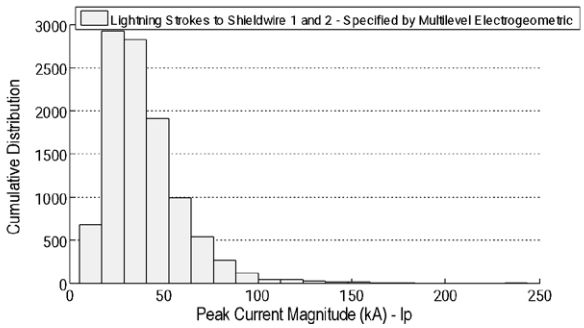


Figure 13. Frequency distribution of lightning peak current on SWs 1 and 2.

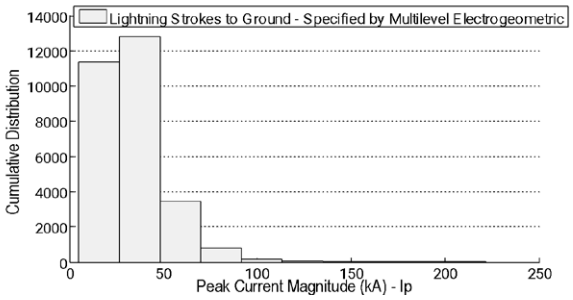


Figure 14. Frequency distribution of lightning peak current on the ground.

It should be mentioned that the TL modeling in ATP/EMTP is the same in all the methods, as described in the previous subsection. However, the annual lightning flashover rate (LFOR) is estimated by:

$$\mathrm{LFOR} = 2 \times \mathrm{N_g} \times \left(\frac{\mathrm{L}}{\mathrm{d_s}}\right) \times \left(\frac{\mathrm{SFFOR+BFR}}{\mathrm{N}}\right) \qquad (25)$$

where L is the TL length *(= 80 km)*, d_s is the span length *(=0.45 km)*, N is the total number of simulations, and N_g (*=1*) is the ground flash density.

The shielding failure flashover rate (SSFOR) is equal to the total outage resulting from the lightning impact on phase conductors *A*, *B*, and *C*. The back flashover rate (BFR) is also the total outage resulting from the lightning impact on SWs 1 and 2.

Considering the symmetry of the configuration, the calculations are performed for one side of the impact area and, therefore, Equation (25) is multiplied by 2.

Table 2 shows the calculated outputs of each method for the same inputs, such as lightning parameters, impact angle, and striking point terminated on the ground. From Table 2, the number of outages, stroke current magnitude, and resultant outage rate can be separately calculated for each phase conductor and each SW using the proposed MEGM method. However, in the previous methods, the lateral phases have similar shielding failure. Additionally, the number of impacts, the mean value of overvoltages, and the LFOR calculated by the proposed method are less than the results estimated by the conventional methods. This would affect the protection methodology and protection cost directly.

Because Method 3 ignores the maximum shielding failure current and assumes vertical strokes, all strokes affect the lateral phase conductor or SW. With an increased number of impacts, the mean value of resultant overvoltages on the lateral phase (phase *A*) and the calculated outage becomes higher than the practical values, since many vertical strokes will not hit phase *A*, even though they may be within the striking distance of phase *A*. In Method 2, the number of impacts on the phase conductor is higher compared to the proposed method because the maximum shielding failure current is not considered and the position of the lightning stroke to the ground is fixed. Therefore, the mean value of resultant overvoltages in phase *A* and the corresponding outages are higher than practical values.

However, it should be mentioned that the accuracy and efficiency of the proposed method (i.e., Method 1) are confirmed by comparing the associated results with the field measurement presented in (Visacro et al., 2005).

The proposed MEGM demonstrates a 60.6% reduction in the predicted lightning flashover rate (LFOR = 1.80) compared to the conventional non-vertical EGM (Shafaei & Shariatinasab, 2012) (LFOR = 4.57), and a 46.7% reduction compared to the vertical-stroke EGM (Shariatinasab et al., 2009a) (LFOR = 3.38). This significant improvement directly translates to more accurate protection planning and cost savings.

Table 2. The calculated outputs of the three methods.

Lightning & Electrical Parameters	Method 1 (Proposed Method)	Method 2 (Shafaei & Shariatinasab, 2012)	Method 3 (Shariatinasab et al., 2009)
Number of strokes hitting phase A	548	736	1506
Number of outages caused by strokes to phase A	118	192	354
Max. and min. discharge current magnitude on phase A (kA)	9-59	14-179	6-61
Mean value of overvoltage of phase A (kV)	2815.29	3149.02	3885.61
Number of strokes hitting phase C	470	1428	This method cannot distinguish strokes to phase A or C
Number of outages caused by strokes to phase C	73	264	
Max. and min. discharge current magnitude on phase C (kA)	6-61	9-132	
Mean value of overvoltage of phase C (kV)	2641.13	3520.9	
Number of strokes hitting SW 1	7064	- 8456 strokes hit both SWs (cannot be separated by this method) - 58 outages are countered	11576
Number of outages caused by strokes to SW 1	7		26
Max. and min. discharge current magnitude on SW 1 (kA)	83-244		7-222
Number of strokes hitting SW 2	3284		This method cannot distinguish strokes to SW1 or SW2
Number of outages caused by strokes to SW 2	5		
Max. and min. discharge current magnitude on SW 2 (kA)	5-173		
Number of total outages for 40000 lightning strokes	203	514	380
Annual LFOR	**1.80**	4.57	3.38

5.3 Validation with Field Data

During the development phase of the proposed MEGM model, its performance was cross-verified against historical lightning outage records from a segment of the Iranian 400 kV transmission network. The results demonstrated a close agreement between the model's predictions and the actual field data, providing strong, practical validation for its accuracy. While these specific field records are subject to utility confidentiality agreements and cannot be published in full detail—a common limitation in power engineering research—this exercise significantly bolstered confidence in the model's real-world applicability. The validation presented in this paper, through comparison with established simulation models and the available field data from (Visacro et al., 2005), aligns with this initial verification and confirms the model's robustness.

6. Conclusion

The proposed MEGM offers a balanced trade-off between accuracy and computational efficiency. While leader progression models (LPMs) are recognized for their higher theoretical accuracy in simulating the physical progression of a lightning leader, they are computationally intensive and require detailed parameters that are often not available for large-scale studies. In contrast, the MEGM provides a substantial improvement in accuracy over conventional EGMs, as demonstrated in the results, while remaining computationally efficient and practical for engineering applications such as insulation coordination, risk assessment, and protection strategy optimization for extensive transmission networks.

In conclusion, the key novelty of the MEGM lies in its multi-level logic that provides a discrete assessment

for each conductor, a significant advancement over the collective shielding failure analysis in (Shafaei & Shariatinasab, 2012) and the parameter-optimized but single-level approach of (Hashemian et al., 2018).

From an economic perspective, the improved accuracy of the MEGM offers tangible benefits for utility planning. For the 400 kV test line, adopting the MEGM over the conventional EGM (Shafaei & Shariatinasab, 2012) reduces the predicted lightning outage rate from 4.57 to 1.80 outages per year—a 60% improvement in accuracy. This reduction directly translates to cost savings by preventing over-investment in protective devices such as surge arresters. Based on typical equipment and maintenance costs, utilities could achieve savings in the range of $30,000 to $50,000 per kilometer of transmission line while maintaining or even enhancing system reliability.

This study presents an accurate and improved method to evaluate the lightning performance of the transmission line and resultant outages. The proposed method can precisely determine the termination point of impact of non-vertical strokes to transmission lines (a phase conductor, an SW, or the ground).

To address this, a new multi-level EGM (MEGM) is proposed, which considers all the useful parameters for calculating lightning-related overvoltage, such as the maximum shielding failure current, the termination point of impact, and the impact angle.

Unlike the single-level EGM conventionally used in the literature, the proposed MEGM consists of two levels of five modes and can distinguish the strokes to each of the phase conductors and shield wires separately. Therefore, the MEGM improves the accuracy of the prediction of lightning outage rates of transmission lines.

The selection of the appropriate level/mode for simulation depends on the lightning current magnitude compared to the maximum shielding failure current and the coordinates of the lightning occurrence on the ground surface. The mentioned parameters are defined in each iteration of the Monte Carlo simulation.

Comparing the results of the proposed and conventional methods reveals that the transmission line's outage rate calculated by each method is different. This diversity in the determination of the outage rate influences the selected protection strategy of power utilities from technical and economic points of view. However, the specified outage rate of the presented method is lower than that of the conventional techniques, which means that a lower cost is predicted for achieving standard protection for transmission lines.

A potential limitation lies in the reliance on analytical formulas for parameters like the maximum shielding failure current. Enhancing the model with field-recorded data from monitored transmission lines is a recommended direction for future work.

Appendix A: Sensitivity to Lightning Parameter Distributions

A key strength of the proposed MEGM framework is its adaptability to different input parameter distributions. In this context, a sensitivity analysis was conceptualized. The core innovation of the MEGM lies in its multi-level geometric reasoning for non-vertical strokes. Altering the input distributions would change the statistical weight of different strokes but would not fundamentally alter the model's ability to correctly determine the point of impact for a given set of parameters (current, location, angle). Therefore, while the absolute values of the LFOR might shift slightly with different distributions, the relative improvement of the MEGM over conventional EGM methods—which is the central conclusion of this work—is expected to remain significant and robust. This conceptual robustness underscores the model's utility, and incorporating advanced, region-specific distributions is a recommended direction for future work.

Conflict of interest

The authors declare that they have no conflicts of interest to disclose.

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