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Original

Delay-dependent stability analysis for nonlinearly perturbed and parametric uncertain heat exchanger system with time varying-delay

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Abstract: Delay-dependent stability of network-controlled temperature control system (with feedback loop delay in the presence of time varying delays, exogenous load disturbance and parametric uncertainties) had been chosen as a task and the system was investigated by theoretical analysis. The Lyapunov-Krasovskii functional technique, expressed in LMI framework based stability criteria was used as a tool in this research. The inevitable time-delays, unknown exogenous load disturbance and parametric uncertainty are affecting the state evolution of the proposed system. The load disturbance temperature control was mathematically modeled as norm-bounded, nonlinear time varying function of current and delayed state vectors. The parametric uncertainty was modeled by using Taylor series expansion. Then, the parameters were incorporated into the delay-dependent stability analysis. In addition to the minimal number of decision variables, tighter bound integral inequality, using reciprocal convex combination lemma was employed in the analysis to obtain less conservative stability criteria. The results obtained are in accordance with the theoretically obtained in the heat exchanger and they are closer to the standard benchmark temperature-control of heat exchanger system.

Keywords: Delay-dependent stability, linear matrix inequality, Lyapunov-Krasovskii functional, nonlinear perturbations, time-varying delays, temperature control, heat exchanger, parametric uncertainty

1. INTRODUCTION

In this fast growing communicative world, network based control/networked control systems (NCSs) units are the default modules in all modern control equipments (Cloosterman, Van de Wouw, Heemels, & Nijmeijer, 2009). These control units include: valves, actuators, sensors, and Processes etc, that are connected through the communication channel (Liu & Li, 2014). In the recent years, the use of NCSs has received more attention due to The stability, flexibility, reliability, cost-effectiveness, maintenance-free and flexible applications et, are some of the prerequisites of the networked control systems (NCSs) (Cloosterman et al., 2009; Liu & Li, 2014). Due to the uses of NCSs for sharing the signal/data in a common network have some limitations such as network-induced time delays (Chung, Ibrahim, Asirvadam, Saad, & Hassan, 2016; Jin, Wang, & Zeng, 2015) and packet

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dropout (Yu, Wang, Chu, & Hao, 2004). The packet dropout is unimportant which means, we receive that all signals (input/output) are communicated in a single packet. Hence, the important issues in NCSs are the time-delays effects in the control loop. These time-delays in the NCS are inevitable, and so proper technology must be identified and incorporated (Chung et al., 2016; Cloosterman, et al., 2009; Jin et al., 2015; Liu & Li, 2014; Yu et al., 2004). The stability analysis of dynamical system with time delays is very essential part to design a controller (Gu, Chen, & Kharitonov, 2003; Malek-Zavareo & Jamshidi, 1987; Naranjo-Montoya, 2015; Oladapo, Balogun, Adeoye, Olubunmi, & Afolabi, 2017; Ramakrishnan & Ray, 2015; Wu, He & She, 2010; Venkatachalam, Prabhakaran, Thirumarimurugan, & Ramakrishnan, 2019). Certain unidentified problems, such as exogenous noise and parametric uncertainty are affecting the stability of the networked control systems (He, Liu, Rees, & Wu, 2007; Li & De Souza, 1997; Lakshmanan, Senthilkumar, & Balasubramaniam, 2011; Park, 1999; Parlakci, 2005; Ramakrishnan & Ray, 2015; Ramakrishnan & Ray, 2016; Venkatachalam & Prabhakaran, 2018). In the stability analysis, incorporated into the uncertainties are stated here (*i*) time-varying nonlinear load perturbations with respect to current $x(t)$ and delayed state vector $x(t-\tau(t))$ and (*ii*) time-varying parametric perturbations in the system. The novelty of the study is basically lies in mathematically modelling of load disturbance as a bounded nonlinear function (He et al., 2007; Lakshmanan et al., 2011; Ramakrishnan & Ray, 2016) and norm bounded type parametric perturbations (Park, 1999). The control engineering community finds it very difficult in solving these mathematical problems. A very few researchers took these challenges and put forward few mathematical models to us (Gu & Niculescu, 2003; He et al., 2007; Lakshmanan et al., 2011; Park, 1999; Parlakci, 2005; Li & De Souza, 1997; Moon, Park, Kwon, & Lee, 2001; Ramakrishnan & Ray, 2015; Ramakrishnan & Ray, 2016; Zhang, He, Jiang, Wu, & Wu, 2014). Delay dependent stability of thermal system with uncertainties being mathematical modelled as a bounded function has not been discussed in the survey. Thermal system is dynamical and depends on the temperature of the working substance (López-Morales & Valdés-Asiain, 2005; Sharma, Gupta, & Kumar, 2011). In process Industries, medical applications, generating power stations etc., are using the thermal control system is also known as temperature control system. If the thermal energy is controlled over a remote control center, then the closed loop thermal control method will be finished through a communication link. In the network based thermal control unit, there is a tendency of the system parameters to drift away from the equilibrium point due to the time delay in the feedback thermal system (Sharma et al., 2011). This fact motivated us to design and study the stability of a heat exchanger system with the fundamentally sufficient parameters and conditions based on Lyapunov-Krasovskii functional method in this research work (Boyd, El Ghaoui, Feron, & Balakrishnan, 1994; Park, Ko, & Jeong, 2011). The following attempts have been carried out systematically and validated on standard benchmark of heat exchanger system:

(i) The problems of delay dependent stability of temperature control system by taking into account the effect of time delays, load disturbance and parametric uncertainty.

(ii) The new delay-dependent stability criteria for temperature control system with time-varying delay, exogenous load disturbance and uncertain parametric using Lyapunov-Krasovskii functional is computed as less conservatism (Park et al., 2011).

2. DYANAMIC MODEL OF TEMPERATURE CONTROL SYSTEM WITH DELAY

Let us consider the nominal time-delay system as

$$\dot{x}(t) = Ax(t) + A_d x(t-\tau(t)), \ \ t > 0 \tag{3}$$

$$x(t) = \emptyset(t), \ \ \forall t \in [-\bar{\tau}, 0], \ \bar{\tau} > 0 \tag{4}$$

The mathematical modeling of heat exchanger system is

$$A = \begin{bmatrix} -\frac{1}{T_V} & 0 & 0 & 0 \\ \frac{K_H}{T_H} & -\frac{1}{T_H} & 0 & 0 \\ 0 & \frac{K_F}{T_F} & -\frac{1}{T_F} & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}; \ A_d = \begin{bmatrix} 0 & 0 & \frac{-K_P K_V}{T_V} & \frac{-K_I K_V}{T_V} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

where, $\tau(t)$ is a time-varying function, representing time-delay with known constant scalars $\bar{\tau}$ and μ satisfying.

$$0 \leq \tau(t) \leq \bar{\tau},\ \dot{\tau}(t) \leq \mu,\ \forall t \geq 0 \quad (5)$$

where, $x(t) \in \mathbb{R}^{4\times 1}$ is the state vector, $A \in \mathbb{R}^{4\times 4}$ and $A_d \in \mathbb{R}^{4\times 4}$ are the constant matrices; and $\emptyset(t)$ is describing the initial condition of the system (Peng & Tian, 2008; Ramakrishnan & Ray, 2015).

2.1 TEMPERATURE CONTROL SYSTEM WITH TIME-VARYING DELAY

Theorem1: For the linear time-delay system considered (3), the scalars $\bar{\tau}$ *and* μ should be greater than zero in order to be globally asymptotically stable for satisfying the condition given in the relation (5). The system was considered with the following linear matrix inequalities (Park et al., 2011).

$$\begin{bmatrix} R & S \\ S^T & R \end{bmatrix} \geq 0;\ P > 0;\ \ Q_i > 0, i = 1,2;\ \ R > 0 \qquad \Pi < 0.$$

where

$$\Pi = e_1 P e_4^T + e_4 P e_1^T + e_1(Q_1 + Q_2)e_1^T + e_2(-(1\mu)Q_1)e_2^T - e_3 Q_2 e_3^T + e_4(\tau^2 R)e_4^T - \begin{bmatrix} e_2^T - e_3^T \\ e_1^T - e_2^T \end{bmatrix}^T \begin{bmatrix} R & S \\ S^T & R \end{bmatrix} \begin{bmatrix} e_2^T - e_3^T \\ e_1^T - e_2^T \end{bmatrix} \quad (6)$$

$$e_1 = [I \quad 0 \quad 0]^T,$$
$$e_2 = [0 \quad I \quad 0]^T,$$
$$e_3 = [0 \quad 0 \quad I]^T,$$
$$e_4 = (Ae_1^T + A_d e_2^T)^T.$$

Solution: The above model with conditions was solved theoretically using the following constructions with LK functional technique.

$$V_1(t) = x^T(t)Px(t) \quad (7)$$

$$V_2(t) = \int_{t-\tau(t)}^{t} x^T(s)Q_1 x(s)ds + \int_{t-\tau}^{t} x^T(s)Q_2 x(s)ds \quad (8)$$

$$V_3(t) = \tau \int_{-\tau}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)R\dot{x}(s)dsd\theta \quad (9)$$

where, $P, Q_i, i = 1,2,$ and R are real symmetric positive definite matrices.Define $\eta(t)=\begin{bmatrix} x^T(t) & x^T(t-\tau(t)) & x^T(t-\tau) \end{bmatrix}^T$.

The time-derivative of the Lyapunov-Krasovskii functional $V_i(t), i = 1,2,3$ along with trajectory of (3) as follows:

The time-derivative of $V_1(t)$ is given by

$$\dot{V}_1(t) = \dot{x}^T(t)Px(t) + x^T(t)P\dot{x}(t),$$

$$\dot{V}_1(t) = \eta^T(t)(e_4 P e_1^T + e_1 P e_4^T)\eta(t) \quad (10)$$

The time-derivative of $V_2(t)$ is given by

$$\dot{V}_2(t) = x^T(t)(Q_1 + Q_2)x(t) - \big(1 - \dot{\tau}(t)\big)x^T \big(t - \tau(t)\big)Q_1 x\big(t - \tau(t)\big) - x^T(t-\tau)Q_2 x(t-\tau) \quad (11)$$

Since $\dot{\tau}(t) \leq \mu < 1, \dot{V}_2(t)$ of (11) is expressed as an inequality as follows:

$$\dot{V}_2(t) \leq x^T(t)(Q_1 + Q_2)x(t) - (1-\mu)x^T\big(t - \tau(t)\big)Q_1 x\big(t - \tau(t)\big) - x^T(t-\tau)Q_2 x(t-\tau) \quad (12)$$

which, in other way to be expressed as follows:

$$\dot{V}_2(t) \leq \eta^T(t)(e_1(Q_1 + Q_2)e_1^T)\eta(t) - \eta^T(t)(e_2(1-\mu)Q_1 e_2^T)\eta(t) - \eta^T(t)(e_3 Q_2 e_3^T)\eta(t) \quad (13)$$

The time-derivative of $V_3(t)$ is given by,

$$\dot{V}_3(t) = \dot{x}^T(t)(\tau^2 R)\dot{x}(t) - \tau \int_{t-\tau}^{t} \dot{x}^T(s) R\dot{x}(s)ds. \quad (14)$$

Now, by using the reciprocal convex combination lemma (refer chapter 1, Lemma 9) (Park et al., 2011), (14) is expressed as follows:

$$\dot{V}_3(t) \leq \eta^T(t)\left((e_4(\tau^2 R)e_4^T) - \begin{bmatrix} e_2^T - e_3^T \\ e_1^T - e_2^T \end{bmatrix}^T \begin{bmatrix} R & S \\ S^T & R \end{bmatrix} \begin{bmatrix} e_2^T - e_3^T \\ e_1^T - e_2^T \end{bmatrix} \right)\eta(t) \quad (15)$$

with the following condition established:

$$\begin{bmatrix} R & S \\ S^T & R \end{bmatrix} \geq 0. \quad (16)$$

Now, by combining the derivative term of $V_i(t)$, we could obtain the following condition:

$$\dot{V}(t) \leq \sum_{i=1}^{3} V_i(t) \tag{17}$$

This condition (17) is expressed quadratically as follows:

$$\dot{V}(t) \leq \eta^T(t) \Pi \eta(t) \tag{18}$$

Now, if the condition $\Pi < 0$, it implies that the temperature control system in (3) satisfying (5) is asymptotically stable by using Lyapunov functional technique.

2.2 ROBUST STABILITY FOR NOMINAL SYSTEM WITH NONLINEAR ERTURBATION

In this section, we consider the delay dependent robust stability of the delayed temperature controlled system under time-varying and nonlinear perturbation. The state-space model of the state-delayed heat exchanger system with nonlinear perturbations is given below (Ramakrishnan & Ray, 2012)

$$\dot{x}(t) = Ax(t) + A_d x\big(t-\tau(t)\big) + f(x(t),t)$$
$$+ g\big(x\big(t-\tau(t)\big),t\big), \tag{19}$$
$$x(t) = \emptyset(t), \ \ \forall t \in [-\bar{\tau}, 0], \qquad \bar{\tau} > 0$$

where, $A, A_d, \tau(t)$ and $\emptyset(t)$ are already defined in Section 3. In (19), the terms $f(x(t),t)$ and $g\big(x\big(t-\tau(t)\big),t\big)$, represent nonlinear perturbations with respect to the current state and delayed state; they are assumed to satisfy $f(0,t) = g(0,t) = 0$ and the following bounding conditions (Parlakçı, 2006):

$$f^T(.)f(.) \leq \alpha^2 x^T(t) F^T F x(t) \tag{20}$$

$$g^T(.)g(.) \leq \beta^2 x^T\big(t-\tau(t)\big) G^T G x\big(t-\tau(t)\big), \tag{21}$$

Satisfying the following norm-bounded conditions:

$$\|f(.)\| \leq \alpha \|x(t)\| + \beta \big\|x\big(t-\tau(t)\big)\big\| \tag{22}$$

where, $\alpha \geq 0$ and $\beta \geq 0$ are the known scalars. A more generalized version of the condition (23), which is used in this paper, is given by

$$f^T(.)f(.) \leq \alpha^2 x^T(t) F^T F x(t)$$
$$+\beta^2 x^T\big(t-\tau(t)\big) G^T G x\big(t-\tau(t)\big) \tag{23}$$

where, F and G are the known constant matrices of approximate dimensions; and the non-negative scalars α and β quantify the magnitude of the load disturbance to the temperature control system. The problem statement is given below:

To develop a robust stability criterion in LMI framework and to ascertain delay-dependent stability of the system (19) subject to the bounding conditions (20) and (21), and satisfying the condition for the single time-delay case (5), using Lyapunov-Krasovskii (LK) functional approach and reciprocally convex lemma (Park et al., 2011; Ramakrishnan & Ray, 2015).

2.2.1 Stability criterion for temperature control system with time delay and nonlinear perturbation

Theorem2: At any time delay, the stability of the temperature control system is predominant for obtaining the proposed output, so that the delay-dependent stability of the temperature control system with nonlinear perturbation is considered as shown in the relation (19).

Solution: The temperature control system is asymptotically stable if there exist real, symmetric, positive definite matrices $P, Q_i, i = 1, 2,$ and R; scalar $\varepsilon \geq 0$; for any dimensions of free matrix S such that the following LMIs hold:

$$\begin{bmatrix} R & S \\ S^T & R \end{bmatrix} \geq 0;$$

$$P > 0; \ \ Q_i > 0, i = 1,2; \ \ R > 0$$

$$\bar{\Pi} < 0.$$

where,

$$\bar{\Pi} = \bar{e}_1 P \bar{e}_5^T + \bar{e}_5 P \bar{e}_1^T + \bar{e}_1(Q_1 + Q_2 + \epsilon\alpha^2 F^T F)\bar{e}_1^T$$
$$+\bar{e}_2(-(1-\mu)Q_1 + \epsilon\beta^2 G^T G)\bar{e}_2^T - \bar{e}_3 Q_2 \bar{e}_3^T - \bar{e}_4(\epsilon I)\bar{e}_4^T$$
$$+\bar{e}_5(\tau^2 R)\bar{e}_5^T - \begin{bmatrix} \bar{e}_2^T - \bar{e}_3^T \\ \bar{e}_1^T - \bar{e}_2^T \end{bmatrix}^T \begin{bmatrix} R & S \\ S^T & R \end{bmatrix} \begin{bmatrix} \bar{e}_2^T - \bar{e}_3^T \\ \bar{e}_1^T - \bar{e}_2^T \end{bmatrix} \tag{24}$$

with

$$\bar{e}_1 = [I \quad 0 \quad 0 \quad 0]^T,$$
$$\bar{e}_2 = [0 \quad I \quad 0 \quad 0]^T,$$
$$\bar{e}_3 = [0 \quad 0 \quad I \quad 0]^T,$$
$$\bar{e}_4 = [0 \quad 0 \quad 0 \quad I]^T,$$

$$\bar{e}_5 = (A\bar{e}_1^T + A_d\bar{e}_2^T + \bar{e}_4^T)^T.$$

In a nut shell, the problem is regarded as the robust stability of model (19) with boundary conditions (20) and (21) and satisfying (5) is to be solved by using Lyapunov-Krasovskii functional technique and reciprocal convex lemma.

2.3 ROBUST STABILITY FOR NOMINAL SYSTEM WITH TIME-DELAY AND PARAMETRIC UNCERTAINTY

A general dynamical system is considered with single time varying delay and parametric uncertainties with the relation and satisfying the conditions as given in (26).

$$\dot{x}(t) = [A + \Delta A(t)]x(t) + [A_d + \Delta A_d(t)]x\big(t - \tau(t)\big) \quad (25)$$

$$x(t) = \emptyset(t), \ \forall t \in [-\bar{\tau}, 0], \ \bar{\tau} > 0.$$

where, $\Delta A(t), \Delta A_d(t)$ are real-unknown but, norm-bounded matrix functions of $A(t)$ and $A_d(t)$, respectively and they are represented as shown in equation (28) (Peng & Tian, 2008).

$$[\Delta A(t) \quad \Delta A_d(t)] = D\,F(t)[E_a \quad E_b] \quad (26)$$

$$F^T(t)F(t) \leq I, \forall t \quad (27)$$

where, $F(t)$ is a real-unknown time-varying matrix (valve variations with respect to time) which are satisfying, and $D, E_a,$ and E_b are the real-known constant matrices that specify how the uncertain parameters of $F(t)$ appear in the nominal system matrices. In specific, for the given scalars, $\tau > 0, \mu > 0,$ the work in the following sequence investigates if system (26)-(28) is robust and stable for any time-delay $\tau(t)$ satisfying the condition.

2.3.1 Stability of Temperature Control system with Time-Delay and Parametric Uncertainty

Theorem 3*:* In the uncertain system with time-delay (25), the given scalars $\bar{\tau} > 0$ and $\mu > 0$, and the proposed heat exchanger system are globally, robustly and asymptotically stable for any time-delay, satisfying (25) and for the admissible uncertainties, (26) and (27) mathematically expressed as shown in relation (28) (Aguilar-López, Acevedo-Gómez, González, Isabel, & Domínguez-Bocanegra, 2008; Li & De Souza, 1997; Park, 1999; Sun, Liu, Chen, & Rees, 2010; Ramakrishnan & Ray, 2012).

$$\begin{bmatrix} R & S \\ S^T & R \end{bmatrix} \geq 0; \ P > 0; \ \ Q_i > 0, i = 1,2; \ \ R > 0$$

$$\tilde{\Pi} = \begin{bmatrix} A^T P + PA + Q_1 + Q_2 - R & PA_d + R - S^T & S^T & A^T\tau^2 R & PD & \varepsilon E_a^T \\ * & -(1-\mu)Q_1 - 2R + S + S^T & R - S^T & A_d^T\tau^2 R & 0 & \varepsilon E_b^T \\ * & * & -Q_2 - R & 0 & 0 & 0 \\ * & * & * & -\tau^2 R & \tau^2 RD & 0 \\ * & * & * & * & -\varepsilon I & 0 \\ * & * & * & * & * & -\varepsilon I \end{bmatrix} < 0 \quad (28)$$

Proof: Let us reconsider the matrix inequality $\Pi < 0$ defined in (6). One shall replace A and A_d with $\Delta A(t) = A + DF(t)E_a$ and $\Delta A_d = A_d + DF(t)E_b$, respectively, in $\Pi < 0$ and rewrite the resulting inequality in the form of nominal and uncertain parts of heat exchanger system as

$$\Pi + \Pi_u + \Pi_u^T < 0 \quad (29)$$

$$\Pi_u^T + \Pi_u = \begin{bmatrix} (DF(t)E_a)^T P + P(DF(t)E_a) & P(DF(t)E_b) & 0 & (DF(t)E_a)^T \times \tau^2 R \\ * & 0 & 0 & (DF(t)E_b)^T \times \tau^2 R \\ * & * & 0 & 0 \\ * & * & * & 0 \end{bmatrix},$$

One can decompose Π_u and express $\Pi_u = HF(t)E$.

$$\Pi + HF(t)E + (HF(t)E)^T < 0 \qquad (30)$$

Substituting (30) into (29), and applying Schur's complement (Moon et al., 2001), the LMI given in (28) is obtained. Thus, the system (25) with admissible uncertainties (26) and (27) is asymptotically stable (Gahinet, Nemirovski, Laub, & Chilali, 1995; Gu, & Niculescu, 2003; Park et al., 2011).

Remark: To solve the stability factor presented in Theorems 1, 2 and 3, they are formatted as constraint optimization problems, as follows:

$$\max_{P,Q_1,Q_2,R} \tau$$

such that

$$\begin{bmatrix} R & S \\ S^T & R \end{bmatrix} \geq 0;\ P > 0;\ Q_i > 0, i = 1,2;\ R > 0$$

$$\Pi < 0.$$

3. RESULTS AND DISCUSSION

The stability of thermal control system with single time varying delay, load disturbance and parametric uncertainty model was constructed by using MATLAB/LMI toolbox (Gahinet et al., 1995). the gains and time constants of the thermal system used in the analysis are shown in Table 1 (Sharma et al., 2011; Veeraragavan, Duraisamy, Murugan, & Krishnan, 2017; Venkatachalam & Prabhakaran, 2018).

Table 1. Parameter values of heat exchanger system.

Parameters	Gain	Time-Constant
Heat exchanger	$K_H = 34$	$\tau_H = 30$
Valve	$K_V = 1.25$	$\tau_V = 3$
Sensor	$K_F = 0.08$	$\tau_F = 2$

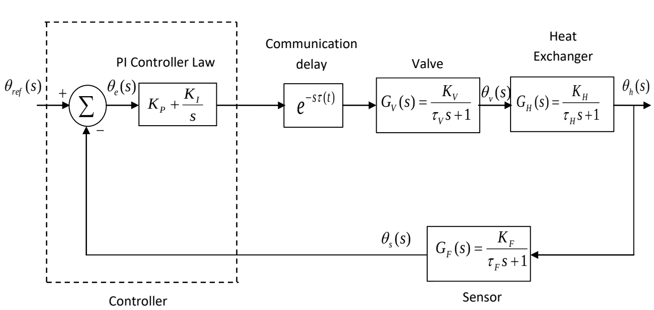

Fig.1. Block diagram of closed loop temperature control with single time delay.

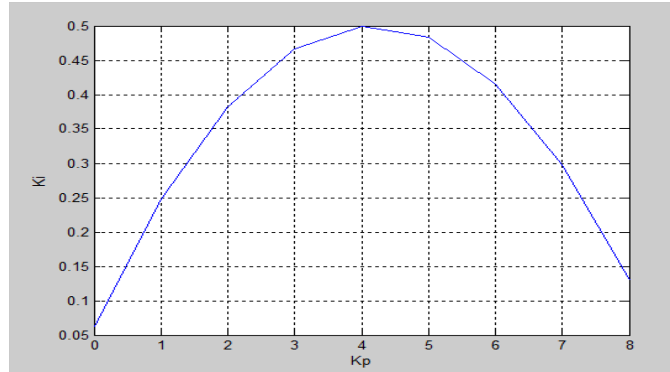

Fig.2. Maximum value of K_I for different values of K_P.

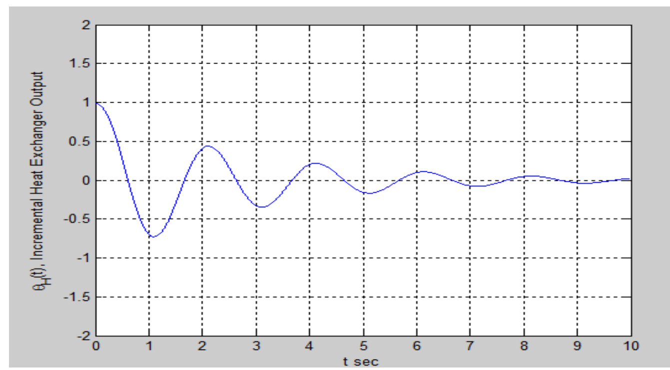

Fig.3. Evolution of $\Delta\theta_H(t)$ for zero delay condition.

For the aforesaid thermal system parameters, with delay-free condition, the system stability curve can be easily drawn; which is illustrated in Fig. 1. From the Fig. 2, it is clear that for all values of gains of controller lying below the stability curve, the closed-loop thermal system is asymptotically stable (Veeraragavan et al., 2017). From the stability curve we can choose the controller parameter approximately for calculate the delay margin. For these controller gains, the closed loop system converges asymptotically to equilibrium point as illustrated in Fig. 3, for a unit step perturbation in temperature control system output variable (from its equilibrium value). When the network delay $\bar{\tau}$ is made zero, the system equation (3) becomes

$$\dot{x}(t) = (A + A_d)x(t) \qquad (32)$$

Now, the Eigen values of the (thermal control) system state matrices are observed $(A + A_d)$ that depend on the PI −controller gains. Table 2 shows that the maximum values of integral controller for a fixed value of proportional controller gain, for which, the thermal control system is on the verge of instability, i.e. having one complex-conjugate pair of eigen values on the $j\omega$

axis. It is observed from the Fig. 2 that there is a tendency for the curve to decrease gradually with increase of K_I.

Table 2. Maximum integral controller gain for a given K_P.

Sl. No:	K_P	K_I	Eigen values of $(A+A_d)$
0	0	0.062	-0.4329±0.009j; -0.0004;-0.0790
1	1	0.248	-0.0000±0.167j; -0.5801;-0.2864
2	2	0.382	-0.0001±0.223j; -0.6415;-0.2250
3	3	0.466	-0.0000±0.267j; -0.6885;-0.1781
4	4	0.499	-0.0000±0.305j; -0.7282;-0.1385
5	5	0.482	-0.0000±0.339j; -0.7631;-0.1036
6	6	0.415	-0.0001±0.370j; -0.7946;-0.0719
7	7	0.298	-0.0000±0.398j; -0.8236;-0.0430
8	8	0.130	-0.0000±0.425j; -0.8506;-0.0160

3.1 TEMPERATURE CONTROL SYSTEM WITH TIME-DELAY

The temperature control system with time delay as stated in the Theorem 1 was executed for various range of PI controller gain (K_P & K_I), and the maximum value of $\bar{\tau}$ were obtained and tabulated (Table3). In the result were $K_P = 0.75, K_I = 0.05, \mu = 0.05$ chosen and obtained the maximum value of time delay $\bar{\tau}$ is 8.6801seconds. It was found that the system becomes unstable, even when the delay margin increases to the value of 9 seconds.

This result clearly brings out the effect of delay on the delay-dependent stability of the temperature control system as shown in Fig.4. Hence, when compared to earlier theorems (Parlakci, 2006; Shao, 2008, 2009) the results presented in this paper portray a more realistic system operating condition and the proposed stability criterion has less number of decision variables compared to the earlier results as given in Table 8.

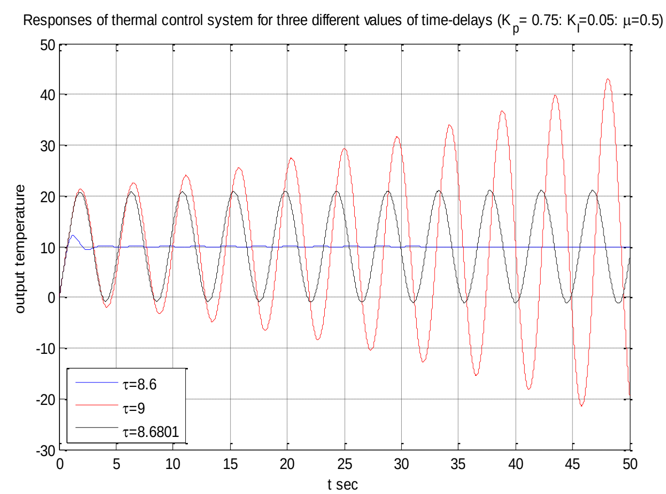

Fig. 4. Step Response to the thermal control system with various delays

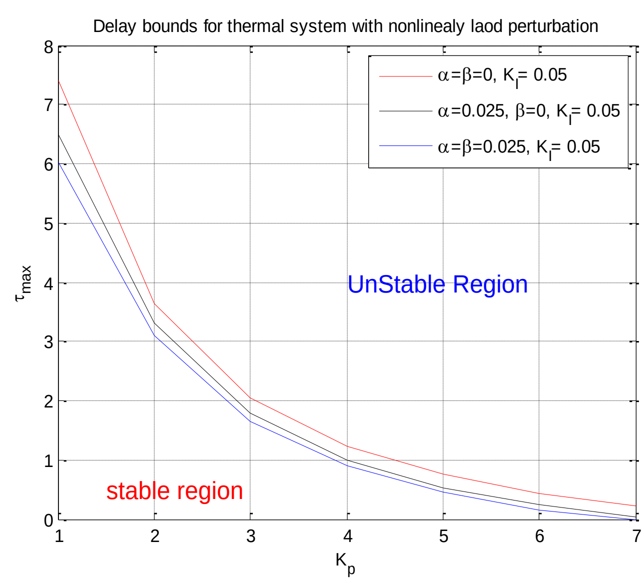

Fig.5 Maximum delay margin for thermal control system with various values of K_P, α *and* β

Table 3. Maximum upper bounds delay for various value of controller parameter and μ.

	Method	$\mu = 0.05$	$\mu = 0.10$	$\mu = 0.20$	$\mu = 0.30$	$\mu = 0.50$
$K_P = 0.75$ $K_I = 0.05$	*Theorem:1* (Parlakci, 2006)	8.6130	8.5109	8.3207	8.1575	7.9829
	corollary:1 (Shao, 2008)	8.6174	8.5292	8.4018	8.3601	8.3601
	corollary:1 (Shao, 2009)	8.6217	8.5474	8.4627	8.3952	8.3914
	Theorem 1	8.6801	8.6427	8.5717	8.5070	8.4299
$K_P = 1.0$ $K_I = 0.05$	*Theorem:1* (Parlakci, 2006)	7.7635	7.6354	7.3847	7.1478	6.7750
	corollary:1 (Shao, 2008)	7.7670	7.6499	7.4491	7.3102	7.3102
	corollary:1 (Shao, 2009)	7.7704	7.6643	7.5117	7.3940	7.3274
	Theorem 1	7.8336	7.7753	7.6619	7.5554	7.4148
$K_P = 1.20$ $K_I = 0.05$	*Theorem:1* (Parlakci, 2006)	6.8127	6.6819	6.4247	6.1800	5.7904
	corollary:1 (Shao, 2008)	6.8157	6.6946	6.4816	6.3247	6.2514
	corollary:1 (Shao, 2009)	6.8188	6.7073	6.5380	6.4091	6.3210
	Theorem 1	6.8792	6.8148	6.6892	6.5714	6.4182
$K_P = 0.75$ $K_I = 0.075$	*Theorem:1* (Parlakci, 2006)	5.6433	5.5905	5.5152	5.4932	5.4934
	corollary:1 (Shao, 2008)	5.6486	5.6127	5.5999	5.5999	5.5999
	corollary:1 (Shao, 2009)	5.6539	5.6326	5.6075	5.6046	5.6046
	Theorem 1	5.6878	5.6726	5.6458	5.6232	5.6099
$K_P = 2.0$ $K_I = 0.01$	*Theorem:1* (Parlakci, 2006)	4.4723	4.3599	4.1403	3.9342	3.6305
	corollary:1 ((Shao, 2008)	4.4745	4.3691	4.1824	4.0434	3.9799
	corollary:1 (Shao, 2009)	4.4766	4.3783	4.2241	4.1076	4.0316
	Theorem 1	4.5231	4.4618	4.3432	4.2336	4.1074
$K_P = 1$ $K_I = 0.15$	*Theorem:1* (Parlakci, 2006)	2.1340	2.1151	2.1121	2.1121	2.1121
	corollary:1 (Shao, 2008)	2.1416	2.1397	2.1397	2.1397	2.1397
	corollary:1 (Shao, 2009)	2.1478	2.1422	2.1402	2.1402	2.1402
	Theorem 1	2.1599	2.1534	2.1434	2.1407	2.1407
$K_P = 3.0$ $K_I = 0.1$	*Theorem:1* (Parlakci, 2006)	1.8700	1.8127	1.7162	1.6580	1.6505
	corollary:1 (Shao, 2008)	1.8729	1.8253	1.7736	1.7721	1.7721
	corollary:1 (Shao, 2009)	1.8758	1.8375	1.7908	1.7799	1.7799
	Theorem 1	1.9039	1.8779	1.8311	1.7933	1.7890
$K_P = 5.0$ $K_I = 0.05$	*Theorem:1* (Parlakci, 2006)	0.7672	0.7338	0.7009	0.7008	0.7008
	corollary:1 (Shao, 2008)	0.7711	0.7506	0.7480	0.7480	0.7480
	corollary:1 (Shao, 2009)	0.7749	0.7593	0.7499	0.7499	0.7499
	Theorem 1	0.7910	0.7759	0.7527	0.7519	0.7519

3.2 TEMPERATURE CONTROL SYSTEM WITH TIME DELAY AND NONLINEAR

In this section, the study corresponding to temperature control system with single time-varying delay, and the exogenous load disturbance effect of the temperature control system is assumed to satisfy the norm-bounded condition given in (23); the both matrices G and F of values are taken as $0.1I_n$ (Ramakrishnan, Venkatachalam, & Ray, 2015) where, n is the system state vector size $x(t)$. Using convex optimization problem stated in remark for Theorem 2, the maximum value of the delay bound τ is calculated for temperature control system for various values of α and β, and for various controller parameter gain settings (viz, K_P and K_I); they are listed respectively, in Tables 4, 5, 6. From the tables, it is understandable that as the values of α and β increases, the load disturbance effects for the temperature control system become more pronounced, and under such perturbed temperature control system conditions, the maximum network delay that the PI controlled temperature control system can withstand, without losing stability decreases. In the result, K_P is varying from $(1-7)$ and $K_I = 0.05, \mu = 0.8$ chosen and obtained the maximum value of time delay $\bar{\tau}$ with various values of load disturbance $\alpha = \beta = 0$; $\alpha = 0.025, \beta = 0; \alpha = \beta = 0.025$ as shown in Fig. 6. This result clearly brings out the effect of load disturbance on the delay-dependent stability of the temperature control system. The results presented in this paper depict a more realistic system operating conditions.

3.3 MAXIMUM ALLOWABLE DELAY BOUNDS FOR TEMPERATURE CONTROL SYSTEM WITH PARAMETRIC UNCERTAINTY

In this section, the temperature control system with parametric uncertainty (parametric variation i.e. time constant of valve is varying with respect to time) are demonstrated and the applications of the Theorem 3 are presented, in order to show the effectiveness of the new robust stability criteria. The maximum value of the delay bound τ is calculated for temperature control system for various values of $\tau_{V_{max}}$, and for various controller parameter gains settings (viz, K_P and K_I); they are listed respectively in Table 7. From the tables, it is clear that as the value of $\tau_{V_{max}}$ increases, the parametric variation effect on the temperature control system become more pronounced as shown in Fig. 6, and under such parametric uncertain temperature control system conditions, the maximum network delay that the PI controlled temperature control system can withstand without losing stability decreases. This result clearly brings out the effect of parametric variations on the delay-dependent stability of the temperature control system. The results present in this paper depict a more realistic system operating conditions.

Table 4. Maximum upper bounds delay $\bar{\tau}$ with load disturbance.

K_P	K_I	$\alpha = \beta = 0$				
		$\mu = 0.05$	$\mu = 0.10$	$\mu = 0.20$	$\mu = 0.30$	$\mu = 0.50$
0.75	0.05	8.6786	8.6412	8.5704	8.5057	8.4288
1.00	0.05	7.8326	7.7743	7.6609	7.5546	7.4141
1.20	0.05	6.8783	6.8139	6.6884	6.5705	6.4174
0.75	0.075	5.6868	5.6718	5.6450	5.6225	5.6092
2.00	0.01	4.5222	4.4608	4.3421	4.2325	4.1062
1.00	0.15	2.1594	2.1528	2.1429	2.1403	2.1403
3.00	0.10	1.9027	1.8767	1.8301	1.7922	1.7880
4.00	0.10	1.3078	1.2868	1.2505	1.2303	1.2303
5.00	0.05	0.7903	0.7752	0.7520	0.7514	0.7514

Table 5. Maximum upper bounds delay $\bar{\tau}$ with load disturbance.

K_P	K_I	$\alpha = 0.025;\ \beta = 0$				
		$\mu = 0.05$	$\mu = 0.10$	$\mu = 0.20$	$\mu = 0.30$	$\mu = 0.50$
0.75	0.05	6.9864	6.9698	6.9389	6.9114	6.8867
1.00	0.05	6.5107	6.4738	6.4026	6.3365	6.2627
1.20	0.05	5.7880	5.7425	5.6543	5.5724	5.4811
0.75	0.075	4.4045	4.3985	4.3883	4.3802	4.3787
2.00	0.01	2.9406	2.2442	1.7986	1.7940	1.7940
3.00	0.10	1.5351	1.5125	1.4736	1.4505	1.4505
1.00	0.15	1.4338	1.4304	1.4270	1.4270	1.4270
4.00	0.10	1.0143	0.9958	0.9658	0.9592	0.9592
5.00	0.05	0.5455	0.5328	0.5227	0.5227	0.5227

Table 6. Maximum upper bounds delay $\bar{\tau}$ with load disturbance.

K_P	K_I	$\alpha = 0.025;\ \beta = 0.025$				
		$\mu = 0.05$	$\mu = 0.10$	$\mu = 0.20$	$\mu = 0.30$	$\mu = 0.50$
0.75	0.05	6.3529	6.3453	6.3355	6.3338	6.3338
1.00	0.05	6.0264	6.0039	5.9623	5.9313	5.9246
1.20	0.05	5.3868	5.3540	5.2963	5.2487	5.2261
0.75	0.075	3.9033	3.9028	3.9027	3.9027	3.9027
1.00	0.15	1.1440	1.1440	1.1440	1.1440	1.1440
3.00	0.10	1.3930	1.3748	1.3501	1.3438	1.3438
4.00	0.10	0.9001	0.8851	0.8681	0.8673	0.8673
5.00	0.05	0.4501	0.4453	0.4420	0.4397	0.4397

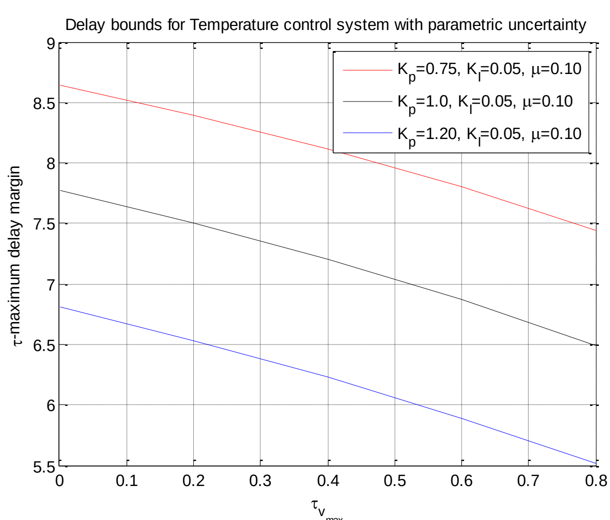

Fig. 6. Maximum delay bounds for thermal control system with various range of uncertainty

Table 7. Maximum upper bounds for parametric variations (valve various with respect to time).

Controller gains	$\tau_{V_{max}} = 0.20$			$\tau_{V_{max}} = 0.40$			$\tau_{V_{max}} = 0.60$			$\tau_{V_{max}} = 0.80$		
	$\mu = 0.10$	$\mu = 0.30$	$\mu = 0.50$	$\mu = 0.10$	$\mu = 0.30$	$\mu = 0.50$	$\mu = 0.10$	$\mu = 0.30$	$\mu = 0.50$	$\mu = 0.10$	$\mu = 0.30$	$\mu = 0.50$
$K_P = 0.75$ $K_I = 0.05$	8.3927	8.2495	8.1700	8.1147	7.9633	7.8819	7.8001	7.6399	7.5575	7.4380	7.2683	7.1861
$K_P = 1.0$ $K_I = 0.05$	7.5035	7.2741	7.1310	7.2051	6.9660	6.8218	6.8718	6.6226	6.4793	6.4926	6.2334	6.0940
$K_P = 1.20$ $K_I = 0.05$	6.5350	6.2834	6.1302	6.2304	5.9708	5.8197	5.8929	5.6257	5.4796	5.5123	5.2384	5.1013
$K_P = 0.75$ $K_I = 0.075$	5.4468	5.3940	5.3812	5.1931	5.1367	5.1249	4.9039	4.8437	4.8336	4.5693	4.5052	4.4978
$K_P = 2.0$ $K_I = 0.01$	4.1833	3.9542	3.8366	3.8881	3.6591	3.5529	3.5698	3.3426	3.2513	3.2213	2.9983	2.9262
$K_P = 1.0$ $K_I = 0.15$	1.9462	1.9344	1.9344	1.7134	1.7029	1.7029	1.4485	1.4403	1.4403	1.1435	1.1387	1.1387
$K_P = 3.0$ $K_I = 0.10$	1.6636	1.5843	1.5843	1.4359	1.3687	1.3687	1.1908	1.1385	1.1385	0.9242	0.8898	0.8898
$K_P = 5.0$ $K_I = 0.05$	0.6003	0.5857	0.5857	0.4171	0.4123	0.4123	0.2293	0.2293	0.2293	0.0338	0.0338	0.0338

Table 8. Number of decision variables.

Theorem's	No of decision variables
Theorem:1(Parlakci, 2006)	$15.5n^2 + 3.5n$
corollary:1(Shao, 2008)	$4n^2 + 4n$
corollary:1(Shao, 2009)	$2.5n^2 + 2.5n$
Theorem 1	$3n^2 + 2n$
Theorem 2	$3n^2 + 2n + 1$

4. CONCLUSION

Temperature control system with time delay, exogenous load disturbance and parametric variations was modeled and constructed, using the tool of simulink in MATLAB software. The non linear perturbed heat exchanger system was formulated and modeled in terms of the current and delayed state vectors. Similarly, the parametric uncertainty was also mathematically formulated, and modeled (using Taylor series expansion) with norm bounded type uncertainties. The stability of these systems was studied systematically by using the Lyapunov-Krasovskii functional method. The reciprocal convex combination lemma was employed in the stability analysis to make it less conservative. All the models were validated with the bench mark of the temperature controller system under different parametric uncertainties. The deduced result of this work is more realistic in operating conditions in real time temperature control system.

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Notations:

$\mathbb{R}^n$ →The set of real n vectors
∈→Belongs to
$\bar{\tau}$ →Maximum time-delay
$t \in [-\bar{\tau}, \quad 0] \to -\bar{\tau} \leq t \leq 0, t \& -\bar{\tau} > 0$:
X^T →Transport of matrix
α & β → Magnitude of load disturbance
μ → Derivative of the time-delay

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