



## Density distribution effect on the analysis of the ${}^6\text{Li} + {}^{58}\text{Ni}$ reaction

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**Abstract:** The effect of density variation on the optical-model (OM) analysis of  ${}^6\text{Li} + {}^{58}\text{Ni}$  is investigated by using bare potentials extracted from different density distributions of the weakly-bound  ${}^6\text{Li}$  nucleus. For each of the bare potentials, the real and imaginary parts of the direct-reaction and fusion polarization potentials are deduced from respective  $\chi^2$  fittings to the elastic scattering and fusion data. The results obtained show that variations in the density distribution of the weakly-bound projectile can change the strengths of the dynamic direct-reaction polarization potential, however, it cannot make a noticeable difference on the OM predictions of fusion cross sections.

*Keywords:* optical model, weakly-bound system, density distribution

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### 1. INTRODUCTION

In the last decades, considerable efforts have been made to analyze nuclear reactions induced by weakly-bound nuclei (Canto, Gomes, Donangelo, & Hussein, 2006; Canto, Gomes, Donangelo, Lubian, & Hussein, 2015; Gomes, Lubian, Padron, & Anjos, 2005; Kolata & Aguilera, 2009). There are different important dynamic and static properties

of weakly-bound nuclei that can strongly affect the fusion mechanism of such systems (Bush, AlKhalili, Tostevin, & Johnson, 1996; Lubian et al., 2007; Gomes, Lubian, & Canto, 2009; Shrivastava et al., 2013; Lukyanov et al., 2017). Compared to the tightly-bound nuclei, the low separation energy of weakly-bound nuclei increases the probability of breakup, which is a typical dynamic feature in the collision of these systems. The static property of weakly-bound nuclei also makes the study of such systems different than that of the tightly-bound systems. A longer tail density distribution of a weakly-bound projectile can affect the bare potential and therefore the reaction mechanism. Considering the difference

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in the predictions of density distributions of a weakly-bound nucleus, it is of great interest to probe the sensitivity of the respective theoretical calculations to the variations of the density distribution. For this purpose, the  ${}^6\text{Li}$  nucleus is selected to investigate the importance of the density distribution in the analysis of weakly-bound nuclei.

The  ${}^6\text{Li}$  nucleus is one of the weakly-bound projectiles which has attracted much attention in the last years (Kukulín, Krasnopol'sky, Voronchev, & Sazonov, 1984; Otomar, Gomes, Lubian, Canto, & Hussein, 2013). Considering the exotic structure of  ${}^6\text{Li}$ , i.e., the extended density distribution and a cluster nature ( $\alpha + d$ ) with a breakup threshold energy of 1.48 MeV, the nuclear reactions induced by this stable, weakly-bound projectile have been the subject of many experimental and theoretical studies (Beck, Keeley, & Diaz-Torres, 2007; Shrivastava et al., 2009; Luong et al., 2013; Aguilera, Martínez-Quiroz et al., 2017; Camacho et al., 2010; So, Udagawa, Kim, Hong, & Kim, 2007). The present work is devoted to a comparative study done by using different density distributions for  ${}^6\text{Li}$  in the optical-model (OM) analyses of  ${}^6\text{Li} + {}^{58}\text{Ni}$ . In these calculations, we probe the effect of variations of the diffuseness parameter of  ${}^6\text{Li}$  on the predictions of the recently measured fusion cross sections for the  ${}^6\text{Li} + {}^{58}\text{Ni}$  system (Aguilera, Martínez-Quiroz et al., 2017). Toward this goal, we calculate three bare potentials for  ${}^6\text{Li} + {}^{58}\text{Ni}$  by the use of different two-parameter Fermi (2pF) density distributions, which include different diffuseness parameters, for  ${}^6\text{Li}$ . Employing these bare potentials in the OM, we derive the appropriate polarization potentials and calculate the fusion cross sections for the selected system. The obtained results are used to discuss the sensitivity of the OM analysis to the density distributions of a weakly-bound projectile.

In the next section, we describe the optical model potentials used, discussing in particular the bare potentials employed in the analysis of  ${}^6\text{Li} + {}^{58}\text{Ni}$ . The results and discussion are presented in Section 3 and finally the conclusions are drawn in Section 4.

## 2. THEORETICAL OUTLINE

To perform the OM analysis for the nuclear reactions, the total potential can be described in the following form

$$U_{TOT} = V_{bare} + V_{Coul} - [iW_{int} + U_F + U_D] \quad (1)$$

projectile and target nuclei and  $V_{Coul}$  is the Coulomb potential with  $r_C = 1.2$  fm. The strong absorption after ions pass through the Coulomb barrier is described by adding a short-range imaginary potential, ( $iW_{int}$ ), to the real part of the interaction. This imaginary potential has a volume Woods-Saxon shape with the depth, radius and diffuseness of  $W_0 = 50$  MeV,  $r_0 = 1.0$  fm,  $a_0 = 0.2$  fm, respectively. In the OM, the effect of breakup on the fusion analysis has been taken into account, in a phenomenological way, by including the dynamic polarization potentials, i.e.,  $U_D(E)$  and  $U_F(E)$ , which account for couplings of the elastic channel to direct-reaction and fusion channels, respectively (So, Udagawa, Kim, Hong, & Kim, 2007; Kim, So, Hong, & Udagawa, 2002b). The polarization potentials are composed of real and imaginary parts as

$$U_{F,D} = V_{F,D} + iW_{F,D} \quad (2)$$

for which, the real and imaginary parts should satisfy the dispersion relation expressed by (Mahaux, Ngo, & Satchler, 1986; Nagarajan, Mahaux, & Satchler, 1985; Satchler & Love, 1979).

$$V(E) = V(E_s) + \frac{E-E_s}{\pi} P \int_0^\infty \frac{W(E')}{(E'-E_s)(E'-E)} dE' \quad (3)$$

In this equation,  $V(E_s)$  shows the value of  $V(E)$  at the reference energy  $E = E_s$  and  $P$  stands for the principal value.

The real and imaginary parts of the direct-reaction polarization potential,  $U_D$ , are included in the OM analysis by using the energy-dependent form as

$$U_D(r, E) = V_D(r, E) + iW_D(r, E) = (V_{D0}(E) + iW_{D0}(E))f'(r) \quad (4)$$

where

$$f'(r) = 4 \times \frac{EXP}{(1+EXP)^2}, \quad EXP = \exp\left[-\frac{(r-R_D)}{a_D}\right] \quad (5)$$

Here,  $R_D = r_D(A_p^{\frac{1}{3}} + A_t^{\frac{1}{3}})$ , and  $r_D$  and  $a_D$  are the reduced radius and diffuseness, respectively. These parameters were obtained by using the experimental elastic scattering angular distributions of  ${}^6\text{Li} + {}^{58}\text{Ni}$  (Aguilera et al., 2009) and the method employed in Ref. (Aguilera, Kolata et al., 2017). The values found in this way are

$r_D=1.61$  fm and  $a_D=0.68$  fm. In addition to  $U_D$ , the energy-dependent form of the fusion polarization potential is also included in the OM with the following form

$$U_F(r, E) = V_F(r, E) + iW_F(r, E) = (V_{F0}(E) + iW_{F0}(E))f(r) \quad (6)$$

where  $f(r) = \frac{1}{(1+\exp[(r-R_F)/a_F])}$ ,  $R_F = r_F(A_P^{\frac{1}{3}} + A_T^{\frac{1}{3}})$ , and the reduced radius and the respective diffuseness are taken as  $r_F = 1.4$  fm and  $a_F = 0.43$  fm (So et al., 2007; Kim et al., 2002b; Kim, So, Hong, & Udagawa, 2002a).

Using the total potential in the optical-model analysis, the fusion cross sections can be obtained from the following expression (Udagawa & Tamura, 1984)

$$\sigma_{fus}(E) = \frac{2}{h\nu} \langle \chi^{(+)} | W_{int} + W_F | \chi^{(+)} \rangle \quad (7)$$

where  $\chi^{(+)}$  represents the distorted wave function that can satisfy the Schrodinger equation with  $U_{TOT}$ .

The density distribution plays a major role in the calculations of the bare potential. Therefore, considering the extended density distribution of the weakly-bound nucleus, the OM analyses of  ${}^6\text{Li} + {}^{58}\text{Ni}$  are carried out through the use of bare potentials obtained from different density distributions of  ${}^6\text{Li}$ .

### 2.1 BARE POTENTIALS USED IN THE OM CALCULATIONS

In order to calculate the bare potential, use was made of the parameter-free Sao Paulo potential (SPP) which is given by Chamon et al. (2002).

$$V_{SPP} = V_{fold}(r) e^{-\frac{4v^2}{c^2}} \quad (8)$$

Here  $v$  defines the relative velocity of the colliding ions,  $c$  is the speed of light, and  $V_{fold}$  is obtained from

$$V_{fold}(r) = \int \rho(r_1) \rho(r_2) V_0 \delta(r - r_1 + r_2) dr_1 dr_2 \quad (9)$$

In this equation,  $V_0 = -456$  MeV fm<sup>3</sup> and  $\rho(r_1)$  and  $\rho(r_2)$  represent the matter densities of the colliding nuclei described by the two-parameter Fermi density distribution as

$$\rho(r) = \frac{\rho_0}{1+\exp(\frac{r-R_0}{a})} \quad (10)$$

The diffuseness and radius are  $a = 0.56$  fm and  $R_0 = 1.31A^{1/3} - 0.84$  fm extracted from a systematic study of densities (Chamon et al., 2002), and  $\rho_0$  is determined from normalization to the nucleus mass. In addition to the diffuseness mentioned, to examine the effect of density variation of  ${}^6\text{Li}$  on the OM analysis of  ${}^6\text{Li} + {}^{58}\text{Ni}$ , we selected two other diffuseness values for the density distribution of  ${}^6\text{Li}$  as well. Considering the extended density distribution of  ${}^6\text{Li}$  and according to the studies done on its density distribution (Antonov et al., 2005; Dobrovolsky et al., 2002), we also used the 2pF density distributions with diffuseness values of 0.59 and 0.63 fm for this nucleus. Using each diffuseness mentioned, i.e.,  $a = 0.56, 0.59,$  and  $0.63$  fm, for the  ${}^6\text{Li}$  density, we calculated the respective bare potential of the selected system. Each  $V_{bare}$  obtained was employed in the OM and the respective appropriate polarization potentials were estimated for the system by using the FRESCO code and its search version, SFRESCO (Thompson, 1988).

### 3. RESULTS AND DISCUSSIONS

The OM analyses and calculations of the polarization potentials for  ${}^6\text{Li} + {}^{58}\text{Ni}$  were done in two steps. First, we used SFRESCO to find the real and imaginary parts of the direct-reaction polarization potential by considering  $U_F=0$ . Based on each of the bare nuclear potentials obtained from different density distributions of  ${}^6\text{Li}$ , we carried out a  $\chi^2$  fitting to the elastic scattering data of  ${}^6\text{Li} + {}^{58}\text{Ni}$  (Aguilera et al., 2009) and extracted the strength parameters of  $U_D$ , i.e.,  $V_{D0}$  and  $W_{D0}$ . The values calculated by minimizing the  $\chi^2/N$  in each of the analyses are shown in Fig. 1(a-c). The dashed curves in this figure demonstrate the results of the dispersion relation, Eq. (3), when the linear approximation shown by the solid line is used for  $W_{D0}(E)$ . As it is seen, the depths found based on all three bare potentials can well satisfy the dispersion relation. From Fig. 1 it is also clear that the use of different bare potentials in the OM analyses leads to different sets of values for the real part of  $U_D$ . More specifically, the OM analysis made by the bare potential derived from the more extended density distribution ( $a=0.63$  fm) results in more negative values for the real part of  $U_D$ . The physical interpretation is that the more diffuse densities actually extend the influence of the attractive nuclear forces to longer distances, which tends to be compensated by more repulsive polarization potentials. The bare potentials and direct reaction polarization potentials

obtained here are used in the second step  $\chi^2$  analysis to calculate the fusion polarization potential for the  ${}^6\text{Li} + {}^{58}\text{Ni}$  system.

Three bare potentials and their corresponding  $U_D$  were independently used in the optical model and for each of them the real and imaginary depths of fusion polarization potential,  $V_{F0}$  and  $W_{F0}$ , were extracted from the simultaneous  $\chi^2$  fitting to the fusion and elastic scattering data of  ${}^6\text{Li} + {}^{58}\text{Ni}$  (Aguilera, Martinez-Quiroz et al., 2017, Aguilera et al., 2009). In these calculations, we kept  $U_D$  found in Fig. 1(a-c) fixed, and treated the real and imaginary depths of  $U_F$  as adjustable parameters. The results showed that, independent of the  $V_{bare} + U_D$  used in the OM analyses, identical values for  $V_{F0}$  and  $W_{F0}$  are obtained. The values extracted are shown in Fig. 2. The result of Eq. (3) using the linear approximation for  $W_{F0}$  is demonstrated by the dashed curve in this figure. As it is seen, the  $V_{F0}$  and  $W_{F0}$  values obtained for  $U_F$  can also satisfy the dispersion relation.

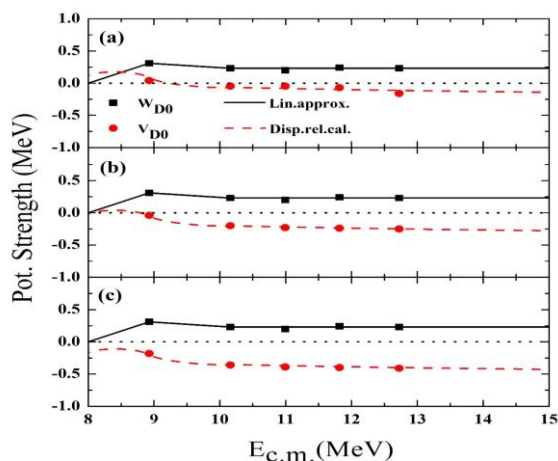


Figure 1. The real and imaginary depths of the direct-reaction polarization potentials obtained by using the bare potentials calculated from the density distributions with (a)  $a=0.56$  fm, (b)  $a=0.59$  fm, and (c)  $a=0.63$  fm.

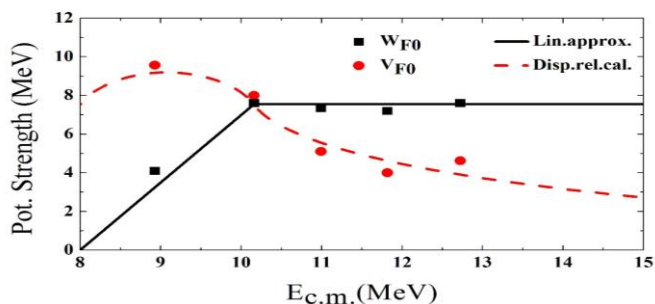


Figure 2. The real and imaginary depths for the fusion polarization potential.

The resulting values of  $U_D$  and  $U_F$ , together with the respective bare potentials, were used to calculate the fusion cross sections of the  ${}^6\text{Li} + {}^{58}\text{Ni}$  system in the OM. The theoretical values obtained are compared with the experimental data in Fig. 3. As it is observed, three different OM calculations done by using the bare potentials derived from different density distributions and their corresponding polarization potentials, result in similar predictions for the fusion cross sections of the weakly-bound system. As it is clear, the cross sections found are in agreement with the experimental data (Aguilera, Martinez-Quiroz et al., 2017) at higher energies. One can also see that the theoretical results cannot reproduce the experimental values at lowest energies. Similar to other  ${}^6\text{Li}$ -induced fusion reactions, such as  ${}^6\text{Li} + {}^{64}\text{Zn}$  (Di Pietro et al., 2013), the disagreement found between the theoretical values and the experimental ones may represent the possibility of contributions of direct reaction channels in the fusion measurements of  ${}^6\text{Li} + {}^{58}\text{Ni}$ . This possibility has been discussed in Ref. (Aguilera, Martinez-Quiroz et al., 2017).

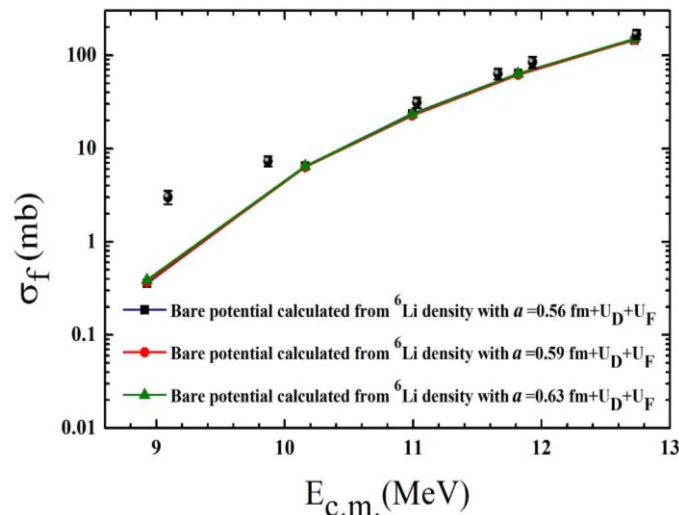


Figure 3. The fusion cross sections obtained from different bare potentials and their corresponding dynamic polarization potentials. The circles show the experimental data which were taken from Ref. (Aguilera, Martinez-Quiroz et al., 2017).

#### 4. CONCLUSIONS

In this study, we examined the sensitivity of the OM analysis of elastic scattering and fusion data for  ${}^6\text{Li} + {}^{58}\text{Ni}$  to the density distribution of  ${}^6\text{Li}$ . Different bare potentials for the selected system were calculated by the use of three  $2pF$  density distributions with different values of the diffuseness parameter for  ${}^6\text{Li}$ . Using the bare potentials



obtained in the  $\chi^2$  analyses of the data resulted in different strengths for the direct polarization potential, but identical predictions for the fusion polarization potential, and similar values for the fusion cross sections of  ${}^6\text{Li} + {}^{58}\text{Ni}$ . Considering the results obtained, one can conclude that the density variations of the weakly-bound projectile in the OM analysis of  ${}^6\text{Li} + {}^{58}\text{Ni}$  can affect the description of the direct-reaction polarization potential and make it more repulsive for more extended density distributions, however, it cannot significantly influence the accuracy of the final fusion results.

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