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Economic production quantity model with backorders and items with imperfect/perfect quality options

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Abstract: In this paper, an economic production quantity (EPQ) model with backorders considering two options for replenishing of items is proposed; with partially imperfect and perfect quality items. First option assumes a produced shipment contains a fraction of imperfect quality items and supplier does not conduct a full inspection. Therefore, these items are detected by a fully perfect screening process by buyer and are sold as a single batch at a discounted price. While the second one assumes that all items that are produced are fully inspected by supplier and all delivered items are perfect; of course with higher unit production price. Ordering size and backordering level are used as decision variables to derive the closed-form optimal solution. The proposed model is illustrated and discussed by a numerical example. Finally, a sensitivity analysis is done for identifying the impact of crucial parameters on the optimal solution.

Keywords: Inventory Control; Economic Production Quantity; Backorders; Imperfect Items

1. INTRODUCTION

The economic production quantity (EPQ) model has been widely used in practice because of its simplicity. However, there are some shortcomings in the assumptions of the basic EPQ model. Two important assumptions of the basic EPQ model are that all the items are of perfect quality and the no shortage constraint. Recently, the classical EPQ model has been generalized in many directions and many researchers have tried to improve it with different viewpoints. Some authors extended the EPQ model by relaxing these restrictions and extend the basic EPQ model to solving inventory models under various realistic situations such as backordering, inflation, varying setup cost, rework process and etc.

Some researchers have studied EPQ model with backordering, including Cárdenas-Barrón (2011), Omar, Zubir, and Moin (2010), Pentico, Drake, and Toews (2009), Sarkar and Moon (2013), Sphicas (2006) and Wee, Huang, Wang, & Cheng (2014). Yang, Teng, and Chern (2010), studied an inventory model under inflation for deteriorating items with stock-dependent consumption rate and partially backordering. Darwish (2008) developed the EPQ model by considering a relationship between the setup cost and the production run length. The effect of

learning and forgetting in setups and in product quality is investigated by Jaber and Bonney (2003). Jaber (2006) investigated the lot sizing problem for reduction in setups with reworks and interruptions to restore the process to an "in-control" state. Afshar and Abbasi (2009) considered an EPQ model with depreciation cost and process quality cost as continuous functions of time. Tettech, Xu, and Liu (2014) studied the effect of speculation in a dual supply chain in controlling cost of inventory. Wee, Wang, and Cardenas-Barron (2013) generalized an EPQ model with rework process at a single-stage manufacturing system with two backordering costs on imperfect manufacturing system. They proposed other decision variables, the production time and the time to eliminate backorders, instead of the classical decision variables of lot size and backorder quantities. Chiu, Tseng, Wu, and Sung (2014) considered a multi-item EPQ with scrap, rework and multiple deliveries. Also, Chiu, Wu,Chiu, and Chang (2015) applied mathematical modelling for determining the optimal production length of the problem.

Many other researchers have studied EPQ model with imperfect production process including Afshar-Nadjafi et al. (2018), Hemmati and Afshar-Nadjafi (2017), Jaber, Zanoni, and Zavanella (2014), Liao and Sheu (2011), Vörös (2013) and Widyadana and Wee (2012). Yoo, Kim, and Park (2009) developed an EPQ model with imperfect production quality, imperfect inspection and rework. Yassine, Maddah, and Salameh (2012) analyzed shipment of imperfect quality items during a single production runs and over multiple production runs. Rezaei and Salimi (2012) studied the relationship between buyer and supplier with regard to conducting the inspection and resulting in a change the buyer's economic order quantity and purchasing price. This work will be the starting point of the model developed in this paper.

To the author's knowledge, none of the above studies considered the economic production quantity (EPQ) model with backorders considering quality-oriented options for replenishing of items. In this paper, the basic EPQ model is extended by considering backorders and two options for replenishing of items; with partially imperfect and perfect quality. A simple method is used to solve the extended model of maximizing the total annual profit. Also, numerical examples are used to show the utility of the proposed models. The organization of the paper is as follows. The model assumptions and notation are presented in Section 2. Then, the model is developed in Section 3. In section 4 a summary of the results given in this paper is explained based on a numerical example. Finally, section 5 contains the conclusions.

2. ASSUMPTIONS AND NOTATIONS

The basic EPQ model is that of determining a production quantity of an item, subject to the following assumptions:

- Demand rate is continuous, known and constant.
- Production rate is greater than or equal to demand rate.
- All demands must be met.
- Holding costs are determined by the value of the item.
- There is no discount.
- There are no quantity constraints.
- All items in a batch conform to quality characteristics.
- No shortages are allowed.

In this section, we derive a mathematical statement for the EPQ model with backordering and quality-oriented options for replenishing of items. Indeed, we consider two scenarios for replenishing of items. First scenario assumes a produced shipment contains a fraction of imperfect quality items and these items are detected by 100% screening by buyer and are sold as a single batch at a discounted price. The second scenario assumes that all produced items are fully inspected by supplier and all delivered items are perfect; of course with higher unit production price. This scenario is the traditional EPQ model in which studied in the literature by relaxing the last assumption (Rezaei & Salimi, 2012).

In the first scenario, we remove the two last assumptions of the basic model considering the following assumptions:

- A shipment contains a fraction of imperfect items.
- Imperfect items are identified via a perfect inspection process
- Shortages are allowed as backlogged.
- Inspection rate is higher than the production rate.

In the second scenario, the same assumptions of the classical economic production quantity as mentioned above are considered. More specifically, it is assumed that

all the items are fully inspected by supplier, therefore all the items are perfect.

In this paper, we examine which scenario is preferred by the buyer?

It is clear that, if the unit price of items is the same, the answer is simply to select the second scenario. However, in reality the unit price of items offered by the supplier in second scenario is higher than the unit price of items in offered by the first scenario, which results in a trade-off between quality and price. To answer the above mentioned question, we need to identify the maximum production price (Mc) the buyer is willing to pay to a supplier belonging to the second scenario. If the unit price is less than Mc, the second scenario is preferred, if not the first scenario is preferred (Chung & Cárdenas-Barrón, 2012).

In order to state the problem mathematically, let:

y production quantity (real positive decision variable)

b the maximal backorder level (real positive decision variable)

D annual demand rate of product

U annual production rate of product

c regular unit production cost of product

c' unit production cost of perfect product

Mc maximum unit production price of perfect product

d inspection cost per unit

s the perfect unit selling price

v the imperfect unit selling price

π the unit fixed backorder cost per unit

$\hat{\pi}$ annual unit backorder cost

h annual unit holding cost

K fixed setup cost of production system

p imperfect rate of production system (random variable)

$E(p)$ expected value of imperfect rate of production system

I_{max} maximum on-hand inventory level

T the length of the inventory cycle

t_1 time to eliminate the backorder position

t_2 time to build an inventory of I_{max} from a zero position

t_3 time to consume the maximum inventory

t_4 time to build a backorder position

$NS(t)$ net stock level at time t

$TP(y,b)$ total profit per ordering cycle

$ETPU(y,b)$ expected annual total profit (objective function of first scenario)

$TPU(y,b)$ annual total profit (objective function of second scenario)

3. MODEL DEVELOPMENT

We depict the proposed inventory system (first scenario) in Fig. 1. As shown in Fig. 1, an inventory cycle, i.e., T, can be split into four time-intervals.
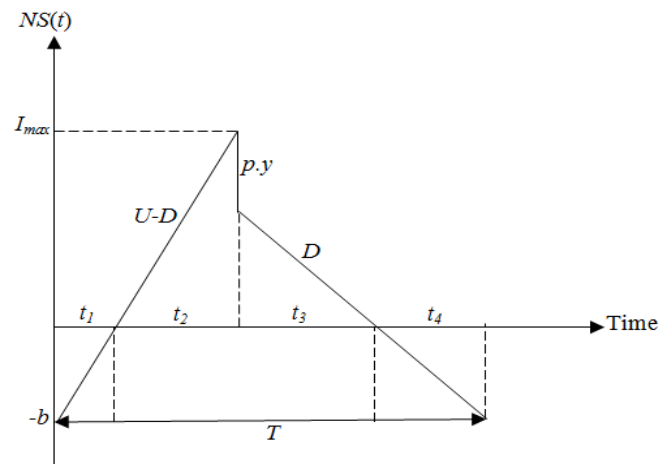

Fig. 1. The relation between net stock level and time.

From graphical representation of model in Fig. 1, we have:

$$t_1 = \frac{b}{U-D} \tag{1}$$

$$t_2 = \frac{y\left(1-\frac{D}{U}\right)-b}{U-D} \tag{2}$$

$$t_3 = \frac{y\left(1-\frac{D}{U}-p\right)-b}{D} \tag{3}$$

$$t_4 = \frac{b}{D} \tag{4}$$

$$I_{max} = y\left(1-\frac{D}{U}\right)-b \tag{5}$$

The buyer's expected profit per ordering cycle is as follows:

$TP(y,b)$ = sales of perfect quality items + sales of imperfect quality items – variable production cost – fixed setup cost - inspection cost - holding costs – shortage costs

Equivalently, we have:

$$TP(y,b) = sy(1-p) + vyp - cy - K - dy - \pi b - \frac{\hat{\pi}b^2}{2D\left(1-\frac{D}{U}\right)} - \frac{h\left(y\left(1-\frac{D}{U}\right)-b\right)^2}{2(U-D)} - \frac{h\left(y\left(1-\frac{D}{U}\right)-b-py\right)^2}{2D} \quad (6)$$

Then the buyer's expected total profit per time unit (annual) can be stated by replacing p by $E(p)$ and dividing it by the expected length of the production cycle $E(T) = \frac{(1-E(p))y}{D}$ as follows:

$$ETPU(y,b) = \left[sD(1-E(p)) + vDE(p) - cD - \frac{KD}{y} - dD - \frac{\pi Db}{y} - \frac{\hat{\pi}b^2}{2y\left(1-\frac{D}{U}\right)} - \frac{hD\left(y\left(1-\frac{D}{U}\right)-b\right)^2}{2y(U-D)} - \frac{h\left(y\left(1-\frac{D}{U}\right)-b-E(p)y\right)^2}{2y} \right] / (1-E(p)) \quad (7)$$

Set $\frac{\partial ETPU(y,b)}{\partial b}$ equals zero, it yields:

$$b^* = \frac{[-\pi D + hy^*(1-E(p))]\left(1-\frac{D}{U}\right)}{\hat{\pi}+h} \quad (8)$$

Substituting b^* in $\frac{\partial ETPU(y,b)}{\partial y} = 0$, we have:

$$y^* = \sqrt{\frac{\frac{2DK(\hat{\pi}+h)}{\left(1-\frac{D}{U}\right)} - \pi^2 D^2}{2h(\hat{\pi}+h)\left((E(p)+0.5) + \frac{E^2(p)}{2\left(1-\frac{D}{U}\right)}\right) - h^2(1-E(p))^2}} \quad (9)$$

The elements of Hessian matrix are:

$$\frac{\partial^2 ETPU(y,b)}{\partial b^2} = \frac{-(\hat{\pi}+h)}{y\left(1-\frac{D}{U}\right)(1-E(p))} \quad (10)$$

$$\frac{\partial^2 ETPU(y,b)}{\partial b \partial y} = \frac{\partial^2 ETPU(y,b)}{\partial y \partial b} = \frac{1}{y^2(1-E(p))}\left[\pi D + \frac{(\hat{\pi}+h)b}{\left(1-\frac{D}{U}\right)}\right] \quad (11)$$

And

$$\frac{\partial^2 ETPU(y,b)}{\partial y^2} = \frac{-1}{y^3(1-E(p))}\left[2D(K+\pi b) + \frac{(\hat{\pi}+h)b^2}{\left(1-\frac{D}{U}\right)}\right] \quad (12)$$

The values α_1 and α_2 can be computed as follows:

$$\alpha_1 = \frac{\partial^2 ETPU(y,b)}{\partial b^2} \leq 0 \quad ; \quad \forall \, b, y \quad (13)$$

$$\alpha_2 = det\nabla^2 ETPU(y,b) = det\begin{bmatrix} \frac{\partial^2 ETPU(y,b)}{\partial b^2} & \frac{\partial^2 ETPU(y,b)}{\partial b \partial y} \\ \frac{\partial^2 ETPU(y,b)}{\partial y \partial b} & \frac{\partial^2 ETPU(y,b)}{\partial y^2} \end{bmatrix} = \frac{(\hat{\pi}+h)}{y^4\left(1-\frac{D}{U}\right)(1-E(p))^2}\left[2DK - \frac{\pi^2 D^2\left(1-\frac{D}{U}\right)}{\hat{\pi}+h}\right] \; ; \; free\ in\ sign \quad (14)$$

Evaluation of α_1 and α_2 shows that the $ETPU(y,b)$ is a non-concave function. Therefore, taking the partial derivatives of $ETPU(y,b)$ with respect to the b and y, and solving equations $\frac{\partial ETPU(y,b)}{\partial b} = 0$ and $\frac{\partial ETPU(y,b)}{\partial y} = 0$ don't guarantee the necessary conditions for b^* and y^* to be optimal. However, we can attain the optimal policy for the first scenario as follows:

If $\quad \pi^2 D^2 \leq \min\{\frac{2DK(\hat{\pi}+h)}{\left(1-\frac{D}{U}\right)}, \frac{2DKh(1-E(p))^2}{(2E(p)+1)\left(1-\frac{D}{U}\right)+E^2(p)}\}$, the optimal solution to maximize $ETPU(y,b)$ is $(b^*, \, y^*)$ obtained from Eqs. (8) and (9), else, the optimal policy is to fill all demand without backorders.

According to Eq. (14) if $\pi^2 D^2 \leq \frac{2DK(\hat{\pi}+h)}{\left(1-\frac{D}{U}\right)}$, we have $\alpha_2 \geq 0$, which guarantees the concavity of $ETPU(y,b)$. Also, by substituting Eq. (9) in Eq. (8) and setting $b^* \geq 0$, we have $\pi^2 D^2 \leq \frac{2DKh(1-E(p))^2}{(2E(p)+1)\left(1-\frac{D}{U}\right)+E^2(p)}$ which guarantees feasibility (non-negativity) of b^* obtained from Eq (8). If we have $\pi^2 D^2 > \frac{2DKh(1-E(p))^2}{(2E(p)+1)\left(1-\frac{D}{U}\right)+E^2(p)}$, we get $b^* < 0$, then (y^*, b^*) is never the optimal solution of $ETPU(y,b)$ on $y > 0$ and $b \geq 0$. So, if the optimal solution of $ETPU(y,b)$ on $y > 0$ and $b \geq 0$ exist, then, $b^* = 0$.

Notice that, in special case of $\pi = 0$, concavity of $ETPU(y,b)$ and feasibility (non-negativity) of b^* is guaranteed. In this case, that b^* and y^* are computed as follows:

$$b^* = \frac{hy^*(1-E(p))\left(1-\frac{D}{U}\right)}{\hat{\pi}+h} \quad (15)$$

$$y^* = \sqrt{\frac{2DK(\hat{\pi}+h)}{2h(\hat{\pi}+h)\left((E(p)+0.5)\left(1-\frac{D}{U}\right)+0.5E^2(p)\right) - h^2(1-E(p))^2\left(1-\frac{D}{U}\right)}} \quad (16)$$

To verify our model by reducing it to the model with perfect items and backordering not considering fixed backordering cost, we set $\pi = 0$ and $E(p) = 0$. One can show that b^* and y^* are computed as follows which are identical to the results in Chung and Cárdenas-Barrón (2012):

$$b^* = \sqrt{\frac{2DKh\left(1-\frac{D}{U}\right)}{\hat{\pi}(\hat{\pi}+h)}} \qquad (17)$$

$$y^* = \sqrt{\frac{2DK}{h\left(1-\frac{D}{U}\right)}}\sqrt{\frac{\hat{\pi}+h}{\hat{\pi}}} \qquad (18)$$

For second scenario, we assume the unit price (c') paid for fully perfect items to be higher than that of a batch with some imperfect items; $c' > c$. Given fully perfect items $(p = 0)$, which result to zero inspection for the buyer, the total profit per time unit is:

$$TPU(y,b) = (s - c')D - \frac{D}{y}(K + \pi b) -$$
$$\frac{\hat{\pi}b^2 + h\left(y\left(1-\frac{D}{U}\right)-b\right)^2}{2y\left(1-\frac{D}{U}\right)} \qquad (19)$$

In this case, the optimal solution to maximize $TPU(y,b)$ is identical to the EPQ inventory model with linear and fixed backorder costs reported in Chung and Cárdenas-Barrón (2012):

$$b^{**} = \frac{[hy^* - \pi D]\left(1-\frac{D}{U}\right)}{\hat{\pi}+h} \qquad (20)$$

$$y^{**} = \sqrt{\frac{2DK}{h\left(1-\frac{D}{U}\right)}\frac{(\hat{\pi}+h)}{\hat{\pi}} - \frac{\pi^2 D^2}{h\hat{\pi}}} \qquad (21)$$

Now the maximum purchasing price for full perfect items should be determined. To this end, we first determine the difference between the total profit per time unit when there are no imperfect items and the expected profit per time unit when there are averagely $p\%$ imperfect items in each batch. We consider c' as a variable here.

$$TPU(y^{**},b^{**},c') - ETPU(y^*,b^*) = (s-c')D - \frac{D}{y^{**}}(K +$$
$$\pi b^{**}) - \frac{\hat{\pi}b^{**2} + h\left(y^{**}\left(1-\frac{D}{U}\right)-b^{**}\right)^2}{2y^{**}\left(1-\frac{D}{U}\right)} - ETPU(y^*,b^*) \qquad (22)$$

The buyer accepts to pay more if and only if

$$TPU(y^{**},b^{**},c') - ETPU(y^*,b^*) \geq 0 \qquad (23)$$

Or equivalently:

$$c' \leq s - \frac{1}{y^{**}}(K + \pi b^{**}) - \frac{\hat{\pi}b^{**2} + h\left(y^{**}\left(1-\frac{D}{U}\right)-b^{**}\right)^2}{2Dy^{**}\left(1-\frac{D}{U}\right)} -$$
$$\frac{ETPU(y^*,b^*)}{D} \qquad (24)$$

The right-hand side of this equation determines the maximum unit purchasing price (Mc) the buyer is willing to pay for batches without imperfect items.

Now, we provide the following rule to answer the research question of this paper: if the item price for the second scenario is less than or equals to the maximum purchasing price Mc computed from Eq. (24), it is wise to select the second scenario (who carries out a full inspection and there are no imperfect items), otherwise the first scenario (here there are averagely $p\%$ imperfect items in each batch and the inspection is conducted by the buyer) is preferred.

4. NUMERICAL EXAMPLE AND DISCUSSION

To illustrate the model developed in section 3, the inventory situation with following data is considered: $U = 12,000$ unit/year, $D = 10,000$ units/year, $K = 450$ \$/cycle, $\hat{\pi} = 0.5$ \$/unit/year, $\pi = 1.2$ \$/ unit, $h = 75$ \$/unit/year, $d = 5$ \$/ unit, $s = 220$ \$/ unit, , $v = 30$ \$/ unit, $c' = 125$ \$/ unit, $c = 100$ \$/ unit and $E(p) = 6\%$. Using these data for the first scenario gives $y^* = 1691.17$, $b^* = 236.71$ and $ETPU(y^*,b^*) = 1,010,030.28$ whereas with the second scenario, we have $y^* = 10241.09$, $b^{**} = 1669.06$, $TPU(y^{**},b^{**}) = 947,165.47$ and $Mc = 110$. It is clear that the first scenario is preferred in which there are averagely 15% imperfect items in each batch and the inspection is conducted by the buyer. However, the optimal backorder level, b^*, and the optimal order quantity, y^*, are decreased in comparison with the second scenario. In this example $Mc = 110$ means that if the item price for the second scenario is less than or equals to $Mc = 110$, this scenario will be preferred.

In order to investigate the effect of imperfect rate of production system, p, the sensitivity analysis is performed by changing $E(p)$. The remaining parameters are kept unchanged.

Table 1. Optimal solutions for different values of E(p).

$E(p)$	y^*	b^*	$ETPU^*$	Mc
0	10241.09	1669.06	1147165.47	105
0.02	2841.45	434.54	1130996.68	106.6
0.04	2053.14	299.84	1114372.28	108.3
0.06	**1691.17**	**236.71**	**1097141.96**	**110**
0.08	1472.65	197.82	1079215.51	111.8
0.1	1322.65	170.59	1060524.62	113.7
0.12	1211.63	150.04	1041005.39	115.6
0.14	1125.24	133.73	1020592.83	117.7
0.16	1055.59	120.31	999218.31	119.8
0.18	997.93	108.99	976808.08	122
0.2	949.20	99.23	953281.95	124.4
0.22	907.33	90.68	928552.19	126.9
0.24	870.87	83.09	902522.27	129.5
0.26	838.75	76.27	875085.57	132.2
0.28	810.20	70.09	846123.85	135.1
0.3	784.61	64.44	815505.57	138.2

Table 1 gives the optimal solutions for selected values of $E(p)$ ranging from 0 to 0.30. The results show that the economic production quantity, economic backordering level and the optimal buyer's expected total profit per time unit (annual) decrease as $E(p)$ increases, while Mc is an increasing function of $E(p)$. These results are demonstrated in Figs. 2, 3 and 4. Fig. 2 shows that as $E(p)$ increases, optimal solution goes to infeasibility (negativity of b^*). In Fig. 2, the initial point, $E(p) = 0$, is the optimal solution related to the supplier with full perfect items (second scenario). It can be observed that the supplier with full perfect items is always characterized by a higher lot size and higher backordering level compared to the supplier with partially imperfect items (first scenario).
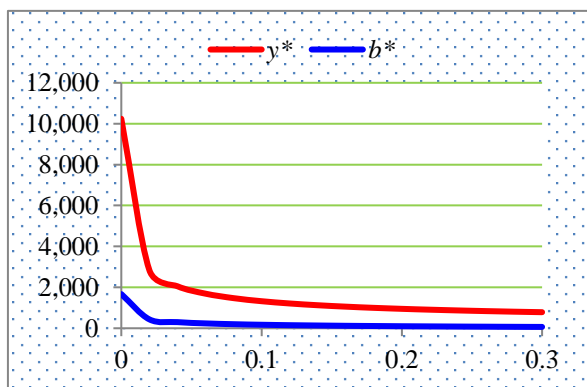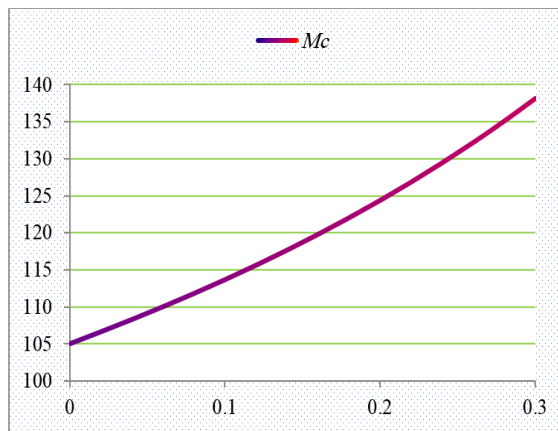

Fig. 2. Optimal values of y^*

Fig. 3. Optimal values of Mc against $E(p)$.

Fig. 3 shows that as $E(p)$ increases, the buyer is willing to pay more to a supplier belonging to the second scenario to avoid receiving imperfect items. Paying this additional amount implies that the supplier conducts the inspection process. Fig. 4 shows that for low values of $E(p)$ the supplier belonging to the first scenario is preferred. However, as $E(p)$ increases, the profitability of the first scenario decreases rapidly. In our example for $(p) > 20\%$, second scenario is preferred.
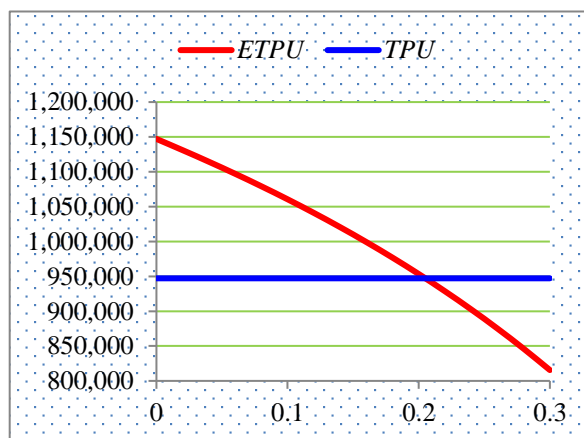

Fig. 4. Comparison of two scenarios with different values of $E(p)$.

An interesting analysis can be carried out by considering changing the purchasing cost of an perfect item, i.e., C', while keeping all other parameters unchanged. The result is reported in Fig. 5, where it can be seen that for values of C' smaller than $Mc = 110$ it is more profitable to select a supplier belonging to the second scenario, otherwise it is more profitable to select first one.
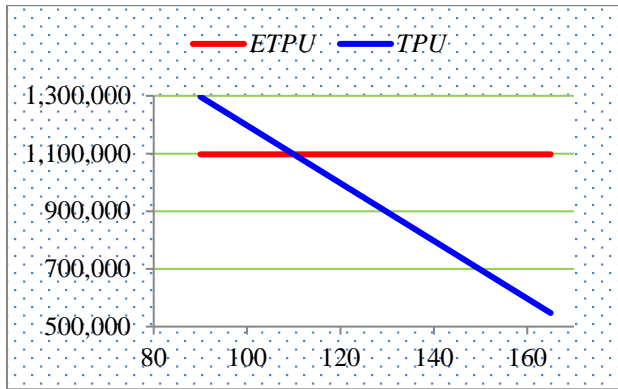
Fig. 5. Comparison of two scenarios with different values of **C′**.

5. CONCLUSIONS

In this paper, a variation of the EPQ model have been developed considering backorders and two options for replenishing of items; with partially imperfect and perfect quality. The option with perfect quality items is simply the traditional EPQ with backorder in which studied in the literature. In this paper, we have formulated and solved the option with imperfect quality items to determine the economic production quantity, economic backordering level and maximum purchasing price a buyer is willing to pay to a supplier to avoid receiving imperfect items. The problem described with a mathematical model, and the existence of the optimal policy is investigated. Also, taking the limiting parameter values for the optimal solutions, our model can be generalized to the classic EPQ model with backorders and perfect items. The numerical example is provided to illustrate the developed model. From the numerical results, we could clearly see that loss due to using the classical EPQ model with backorders is significant. The sensitivity analysis showed that the profitability of a supplier with partially imperfect quality items decreases with increase in the value of the purchasing cost of a perfect item. Also, the results showed that the imperfect rate of production system, p, has significant effect on optimal policies. The results of this study can help managers make optimal decisions on supplier selection. Further research can investigate the case in which there is rework option for imperfect items and shortages are partially backordered.

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