CONTROL OF A SADDLE NODE BIFURCATION IN A POWER SYSTEM USING A PID CONTROLLER

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ABSTRACT

In this work, we present the elimination of a saddle-node bifurcation in a basic power system using a PID controller. In addition, a stability analysis of the rotor angle and its frequency, which are directly related to voltage collapse problem, is presented.

RESUMEN

En este trabajo se presenta la eliminación de una bifurcación de un nodo tipo “Saddle” en un sistema de potencia básica utilizando un controlador PID. Asimismo se presenta el análisis de estabilidad del ángulo del rotor y de su frecuencia que están directamente relacionadas con el problema de colapso del voltaje.

KEYWORDS: Saddle-node bifurcations, PID controller, power systems

1. INTRODUCTION

Voltage collapse in electric power systems has recently received significant attention in the literature. This has been attributed to increases in demand that result in operation of electric power system close to its stability limits. In several papers, for example [1, 2, 3, 4, 5], voltage collapse is related to a saddle-node bifurcation of an equilibrium point, which occurs when the real or reactive power demand is quasi-statically varied. Some other types of bifurcations present in a voltage collapse are Hopf, period-doubling, and cyclic bold types; furthermore, the system may display even chaotic behavior. Several previous works have proposed some procedures to control these bifurcations [1,7,9]; however, they have been focused on bifurcation control of periodic solutions.

Voltage collapse is a system instability that involves many power system components and their variables. This phenomenon often involves the entire power system. Indeed, the rotor angle of the machine is the main variable involved in a voltage collapse. For this reason, there is not difference between a voltage collapse, angle collapse and classical instability.

The main difference between a voltage collapse and classical transient stability is that the voltage collapse focuses on loads and voltage magnitudes, whereas classical transient stability focuses on generators and angles. In addition, voltage collapse often includes longer time scale dynamics as well as the effects of discrete events such as line outages.

A voltage collapse can be seen as a saddle-node bifurcation of equilibrium points. A saddle-node bifurcation is a disappearance of equilibrium points as a parameter value changes slowly. The saddle-node bifurcation of most interest occurs when the equilibrium point where the power system operates disappears. As a saddle-node bifurcation can produce a voltage collapse, it is useful to study this phenomenon to understand and avoid it.

In this way, the work reported in this paper presents the elimination of a saddle-node bifurcation in a basic power system proposed in [6] using a classical PID controller. We found the conditions that the system and the controller must satisfied such that this elimination can be achieved. Also, we analyze the stability of the rotor angle and its frequency, which are directly related to the voltage collapses. In this way one can expect that the results obtained here could be extended to more general systems displaying these undesirable phenomena.

2. A POWER SYSTEM MODEL

The equation that describes the rotor motion of a synchronous generator is

$$J\ddot{\theta} = T_a \quad N \cdot m, \tag{1}$$

where J is the equivalent inertial momentum of all the loads attached to the rotor, θ is the mechanical angle of the shaft, measured with respect to a static framework, and T_a is the resultant torque driving the axis. The machine is a generator, so the driven torque T_m is mechanical and the reacting torque is an electric torque, so that

$$T_a = T_m - T_e.$$

A positive mechanical torque accelerates the rotor, while a positive reacting torque decelerates the machine. If we consider a synchronous rotating reference framework moving with a constant speed ω_R, the angle θ can be expressed as

$$\theta = (\omega_R t + \alpha) + \delta_m,$$

where α is a constant and δ_m is the angular difference between the mechanical angle and the moving frame. Hence, equation (1) is transformed to

$$J\ddot{\delta}_m = J\dot{\omega}_m = T_m - T_e, \tag{2}$$

where ω_R is the derivative of δ_m. By multiplying both sides of (2) by ω_R we obtain a description in terms of power, that is

$$\omega_m J\ddot{\delta}_m = \omega_m T_a - \omega_m T_m - \omega_m T_e = P_m - P_e, \tag{3}$$

where $J\omega_R$, denoted by M, is called the inertia constant, and P_m and P_e are the mechanical and electrical power, respectively.

Let us consider a system composed by two simple generators connected as shown in figure (1), the power supplied by the source $E\angle\delta_m$ is given by

$$P_e = \frac{EV}{X} \sin \delta_m ,$$

where X is the load.

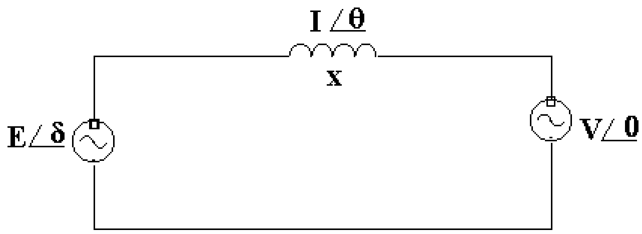

Figure 1. A basic power system.

If a dissipation component $D\omega$ inherent to the machine is added, then the model of this basic power system is given by

$$\begin{aligned} \dot{\delta} &= \omega \\ \dot{\omega} &= \frac{1}{M}\left[P_m - \frac{EV}{X}\sin(\delta) - D\omega \right] \end{aligned} \tag{4}$$

3. A SADDLE-NODE BIFURCATION

From equation (4), we see that the equilibrium point is given by

$$\omega_o = 0, \quad \delta_o = \arcsin \frac{P_m X}{EV} \tag{5}$$

If we take as parameter the reactance X, it can seen directly from this expression that the system has a saddle-node bifurcation at the points $(X,\delta)=(\pm 1, \pm\pi/2)$.
The Jacobian matrix of (4), evaluated at the equilibrium point (5) is

$$A = \frac{\partial f}{\partial x} = \begin{bmatrix} 0 & 1 \\ -\frac{EV}{MX}\cos\left(\arcsin \frac{P_m X}{EV} \right) & -\frac{D}{M} \end{bmatrix} \tag{6}$$

which has the eigenvalues

$$\lambda_{1,2} = \frac{D}{M}\left(-1 \pm \sqrt{1 - \frac{4MEV}{D^2 X}\sqrt{1-\left(\frac{P_m X}{EV}\right)^2}} \right) \tag{7}$$

From Eq. (7) it can be seen that a zero eigenvalue exist when $X=\pm EV/P_m$; these values are candidates to be bifurcation points.

The existence of the saddle-node bifurcation can be proved analytically with Sotomayor's theorem[8]. This theorem establishes the following conditions to have this kind of bifurcation,

$$\omega^T\left(\frac{\partial f}{\partial \theta}\right) \neq 0,$$

$$\omega^T\left[\left(\frac{\partial^2 f}{\partial x^2}\right)_0 (\upsilon,\upsilon)\right] \neq 0,$$

where υ and ω are egenvectors corresponding to the zero eigenvalue of matrix *A* (equation (6) and A^T, respectively), and the subindex *0* denotes the evaluation at the equilibrium point. They are given by

$$\upsilon = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \omega = \begin{pmatrix} 1 \\ \dfrac{M}{D} \end{pmatrix},$$

from where

$$\omega^T\left(\frac{\partial f}{\partial \theta}\right) = \frac{P_m^2}{VD} \neq 0$$

and

$$\omega^T\left[\left(\frac{\partial^2 f}{\partial x^2}\right)_0 (\upsilon,\upsilon)\right] = \frac{P}{D} \neq 0$$

Therefore, the conditions established by Sotomayor's theorem are satisfied, and the system displays a saddle-node bifurcation at the point

$$\left(\omega_0, \delta_0, X_0\right) = \left(0, \arcsin\left(\frac{P_m V}{EV}\right), \frac{VE}{P_m}\right)$$

4. CONTROL OF A SADDLE-NODE BIFURCATION USING A PID CONTROLLER

Let us consider that the mechanical power is given by $P_m=P_0+v$, where P_0 is a nominal input and v is an adjustment with control purposes that can be expressed as $v=Mu$, therefore the system (4) transforms to

$$\begin{aligned} \dot{\delta}_m &= \omega \\ \dot{\omega} &= \frac{1}{M}\left[P_0 - \frac{V}{X}\sin\delta_m - D\omega\right] + u \end{aligned} \qquad (8)$$

The controller proposed has the structure shown in figure 2, where the input control u is given by

$$u = k_p e + k_d \dot{e} + \int_0^t e dt \qquad (9)$$

where $e=\delta_{ref}-\delta_m$ is the error and δ_{ref} is a constant reference angle.

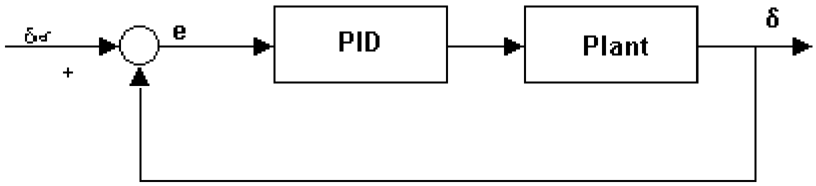

Figure 2. A block diagram of the closed-loop system.

The system (8) can be reduced to a differential equation given by

$$\ddot{\delta}_m = \frac{1}{M}\left[P_0 - \frac{V}{X}\sin\delta_m - D\omega\right] + u \qquad (10)$$

$$\dddot{\delta}_m = \frac{1}{M}\left[-\frac{V}{X}\dot{\delta}_m\cos\delta_m - D\ddot{\delta}_m\right] - k_p\dot{\delta}_m - k_d\ddot{\delta}_m + k_i\left(\delta_{ref} - \delta_m\right) \qquad (11)$$

Now , the system (11) has the new state space form given by

$$\begin{aligned}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= x_3, \qquad (12) \\
\dot{x}_3 &= k_i\left(\delta_{ref} - x_1\right) - \left(k_p + \frac{V}{MX}\cos x_1\right)x_2 - \left(k_d + \frac{D}{M}\right)x_3,
\end{aligned}$$

where $x_1=\delta_m$, $x_2=\omega$ and $x_3=\dot{\omega}$.

4.1 Equilibrium points

From equation (12) we see that the equilibrium points are given by

$$x_1 = \delta_{ref},\ x_2 = 0,\ x_3 = 0.$$

From this expression and using the implicit function theorem, it is possible to see that this is the only equilibrium point; therefore, a saddle-node bifurcation cannot be presented anymore.

5. STABILITY ANALYSIS OF THE CLOSED-LOOP SYSTEM

The Jacobian matrix of the system (12) is given by

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -k_i + \frac{V}{MX}\sin(x_1)x_2 & -\left[k_p + \frac{V}{MX}\cos x_1\right] & -\left[k_d + \frac{D}{M}\right] \end{bmatrix}$$

which, evaluated at equilibrium point $x_1=\delta_{ref}$, $x_2=0$, $x_3=0$, results in

$$A=\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -k_i & -\left[k_p+\dfrac{V}{MX}\cos\delta_{ref}\right] & -\left[k_d+\dfrac{D}{M}\right] \end{bmatrix}$$

This matrix has the characteristic polynomial

$$Z^3+\left(k_d+\frac{D}{M}\right)Z^2+\left(k_p+\frac{V}{MX}\cos\delta_{ref}\right)Z+k_i=0.$$

Let us consider a numerical example given by the values *V=1, D=M=0.5* proposed in [6], and $k_p=k_d=k_i=1$. This leads to the characteristic polynomial

$$Z^3+2Z^2+\left(1+\frac{2}{X}\cos\delta_{ref}\right)Z+1=0.$$

Using the Routh-Hurwitz criteria and, considering that δ_{ref} is limited to $0 \le \delta_{ref} \le \pi/2$, the following stability conditions are obtained

$$\left(1+\frac{2}{X}\cos\delta_{ref}\right)>0 \Rightarrow X\neq 0 \;\&\; X>-2, \qquad (13)$$

$$\frac{2}{X}\cos\delta_{ref}+0.5>0 \Rightarrow X\neq 0 \;\&\; X>-4. \qquad (14)$$

Because *X* is positive, then the conditions (13) and (14) are always satisfied by the closed-loop system.

Figures (3) and (4) show the behavior of the last two expressions, showing that the stability of the closed-loop system does not depend on the parameter values.

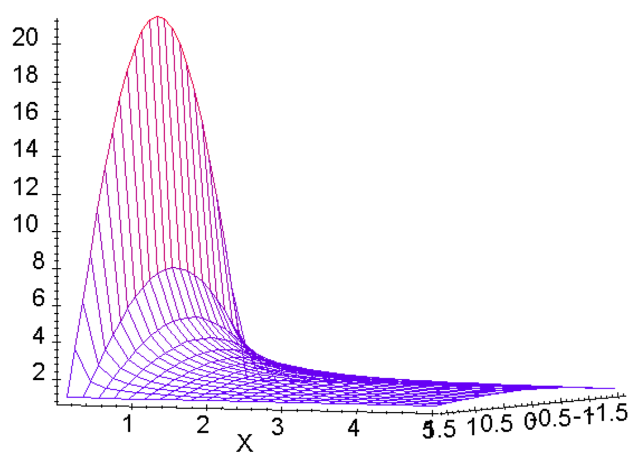

Figure 3. Graph of f = 1 +2/X cosδ_{ref} (condition (13)).

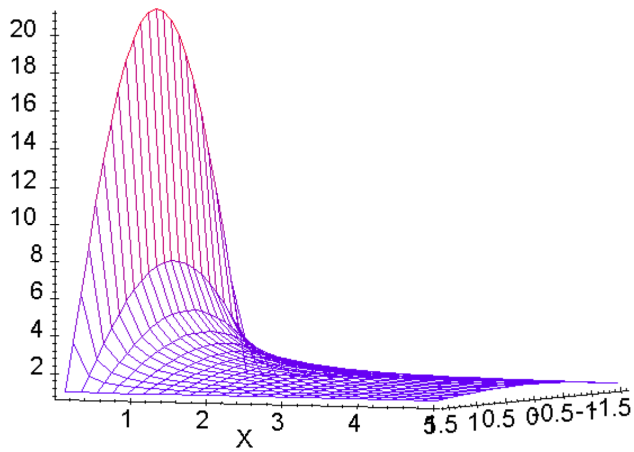

Figure 4. Graph of f = 0.5 +2/X cosδ_{ref} (condition (14)).

Figures (5) and (6) show two numerical simulations, for $\delta_{ref}=0$ and $\delta_{ref}=\pi/2$. We can see that the angle δ_m follows the reference δ_{ref} and $\omega \to 0$ as $t \to \infty$.

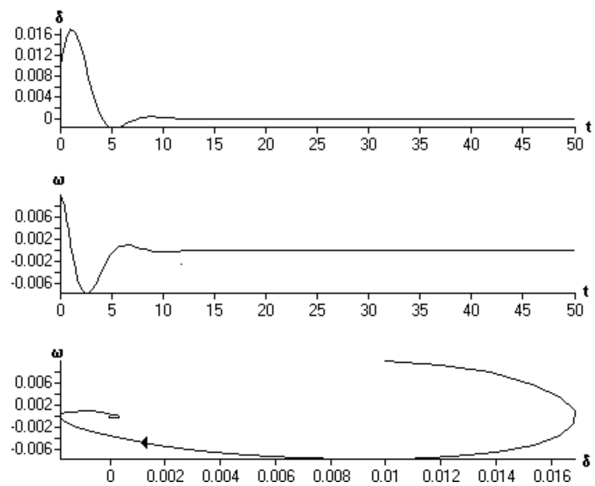

Figure 5. Response of the controlled system for $\delta_{ref}=0$ and X=2.

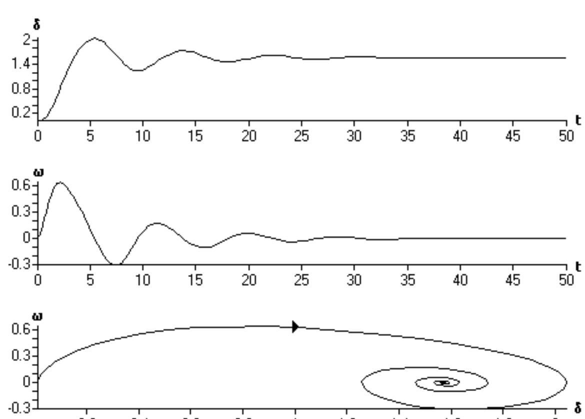

Figure 6. Response of the controlled system for $\delta_{ref}=\pi/2$ and X=2.

6. CONCLUSIONS

In this paper we have analytically proved the existence of saddle-node bifurcations in a simplified model of a power system. This dynamical phenomenon is related to voltage collapses.

We have also proved that the introduction of a simple classical PID controller can eliminate this type of bifurcation, eliminating at least partially the possibility of having collapses in the system.

A power system is very complicated, so we can expect that a real process will present many more complex phenomena, as is indeed the case. The preliminary analysis presented here could be useful, however, to define a systematic way to analyze the conditions to have dangerous behaviors of these important processes, and to envisage some possible form to avoid, or at least to delay these high-risk situations.

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