TIME REVERSIBILITY IN ACOUSTIC SIGNALS

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ABSTRACT

In this work we make a description of the time reversion problem in sound waves. Our objective is to explain the phenomena through the Fourier transform of the Green's function. With this function it is possible to characterize the propagation of the emitted signal. It also can be used to express the time-reversed signal in such a way that we can select precisely the destination site of the signal. Finally, we show some possible practical applications of this problem.

KEYWORDS: Green's Functions, Time Reversal Process, Inverse Scattering Analysis, Acoustics.

1. INTRODUCTION

Time reversibility arises from the second order time derivatives of the wave equation and is different from the phenomena represented by the nonlinear behavior of media, known as inverse acoustic or electromagnetic scattering, studied in other sites [1]. This subject gives rise to an interesting variety of items. One of them is the time reversal of a signal by a sound mirror (TRM). Another may be the possibility to send a message with a precise physical destination. Very interesting and useful applications might be based on the time reversibility of sound waves. In medicine, it is possible to build devices that can destroy a brain tumor by focusing ultrasound waves; undersea communications can be improved; in material and hydrodynamics analysis we can detect small changes in the isotropic properties of the studied media.

The importance of time reversed acoustics is a great motivation for developing a practical generalized treatment of this phenomena, that can be systematically applied to a broad class of scenarios. In such a treatment we obtain expressions which take into account the geometry and nature of the original signal, and that can be used directly to measure some data needed to reverse the time in acoustic signals. The principal features of this work are firstly to obtain an equation, in the discrete case, that relates the Fourier transformed initial signals at the precise emitting sites to the measured Fourier transformed signals at the precise receiving sites through the discrete Fourier transform of the complete Green's function. Then, by using this relation, to describe the detailed procedure involved in both the time reversion and the sending of messages toward specific places without dispersion. The procedure resembles the well-known technique in quantum mechanics collision theory that relates the incoming wave function with the scattered wave by means of the complete Green's function of the problem. In fact, we develop the Neumann series of our discrete problem and calculate the sum in order to obtain our results.

Once we have sketched the formalism involved, we then comment on some possible applications.

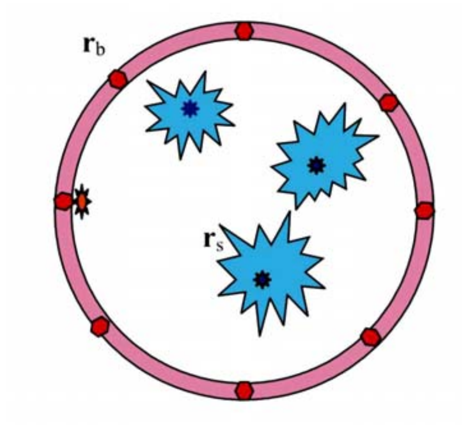

Figure 1. Locus of transducers

2. DESCRIPTION OF THE SYSTEM

We make the assumption that our system is confined to a region of space which contains a set of devices located at $\{\bar{r}_s\}$ (s stands for *site* at the interior). These represent a set of either transmitters or receptors of a sound signal, at our convenience. At the boundary of this region is located another set of devices, at points $\{\bar{r}_b\}$ (b stands for *boundary*), whose operation is identical to the former set, that is, a set of transducers. The transmitters at $\{\bar{r}_s\}$ may be considered themselves as scattering points for the signal generated by other transmitters. If the signal starts at the inner $\bar{r}_s$ as $u_s(t')$, we can record the corresponding signal at location $\bar{r}_b$ as $u_b(t)$ and, by time reversal, recover $u_s(t')$. On the other hand, we could have a problem with a different purpose in which we can emit a signal $u_b(t')$ from a point $\bar{r}_b$ at the boundary and send the message only to the inner site $\bar{r}_s$ as $u_s(t)$. The sound signals are assumed to satisfy the wave equation

$$k(\bar{r})\frac{\partial^2 u(\bar{r},t)}{\partial t^2} = \nabla^2(u(\bar{r},t)/\rho(\bar{r})) \tag{1}$$

Here $\rho(\bar{r})$ is the density and $k(\bar{r})$ is the compressibility of the transmitting medium. This equation is time reversal invariant since the time derivative is of second order. Then, for every $u(\bar{r},t)$ there exists a signal $u(\bar{r},-t)$ that retraces the complex paths and converges in synchrony at the original source. Suppose that a pulse $u(\bar{r},t)$ leaves a source at $\bar{r}_s$; then we can perform physically the time-reversal process by recording by means of the transducers $\{\bar{r}_b\}$ the signal that arrives at the boundary during a time T and thereafter sending the time-reversed signal $u(\bar{r},\mathrm{T}-t)$. Each of the $\{\bar{r}_b\}$ emit the signal $u_b(\mathrm{T}-t)$ also during a time T, so that the resultant field is the original $u(\bar{r},t)$ focused at $\bar{r}_s$.

One may want to make a sort of inverse application to the procedure described above: first we send a message $u_b(\mathrm{T}-t)$ from the boundary at $\bar{r}_b$ and make a similar translation in space and time so the signal arrives at the precise site $\bar{r}_s$ as $u_s(t)$.

Let's carry out these procedures with the Green's function formalism.

3. GREEN'S FUNCTION FOR THE DISCRETE CASE

Suppose we want to relate the values of $u(\bar{r},t)$ at different places and times $\bar{r},t$ and $\bar{r}',t'$. Because of the linearity of the wave equation (1) and remembering that $u(\bar{r}_j,t)=u_j(t)$, it is possible to write [2] for the signal measured at the site $\bar{r}_j$:

$$u_j(t)=u_j^{(\circ)}(t)+\sum_{k\neq j}\int_{-\infty}^{\infty}G^{(\circ)}(\bar{r}_j,t;\bar{r}_k,t')A_k u_k(t')dt' \quad , \tag{2}$$

where $G^{(\circ)}(\bar{r}_j,t;\bar{r}_k,t')$ is the free Green's function and the A_k are complex scattering coefficients that contain the full nonlinear interaction. The signal $u_k(t')$ can be written in terms of the non-vanishing function $S_k(t')$ defined by

$$u_k(t')=\begin{Bmatrix} 0 \quad for \quad t'\in(-\infty,0)\cup(\mathrm{T},\infty) \\ S_k(t') \quad for \quad t'\in[0,\mathrm{T}] \end{Bmatrix} \tag{3}$$

Then, equation (2) can be written

$$S_j(t)=S_j^{(\circ)}(t)+\sum_{k\neq j}\int_{\circ}^{\mathrm{T}}G^{(\circ)}(\bar{r}_j,t;\bar{r}_k,t')A_k S_k(t')dt' \tag{4}$$

But we can express the free Green's function in terms of its Fourier Transform corresponding to the associated frequency ω

$$G^{(\circ)}(\bar{r}_j,t;\bar{r}_k,t')=\frac{1}{2\pi}\int_{-\infty}^{\infty}G_{\omega}^{(\circ)}(\bar{r}_j,\bar{r}_k)e^{i\omega(t-t')}d\omega \tag{5}$$

Thus equation (4) becomes

$$S_k(t)=S_k^{(\circ)}(t)+\frac{1}{2\pi}\sum_{k\neq j} \quad \times\int_{\circ}^{\mathrm{T}}\int_{-\infty}^{\infty}G_{\omega}^{(\circ)}(\bar{r}_j,\bar{r}_k)e^{i\omega(t-t')}d\omega A_k S_k(t')dt' \tag{6}$$

This can be written as

$$S_k(t)=S_k^{(\circ)}(t)+\frac{1}{2\pi}\sum_{k\neq j}A_k\int_{\infty}^{\infty}G_{\omega}^{(\circ)}(\bar{r}_j,\bar{r}_k)\times[\int_{\circ}^{\mathrm{T}}e^{i\omega(t-t')}S_k(t')dt']d\omega \tag{7}$$

Or also

$$S_j(t)=S_j^{(\circ)}(t)+\frac{1}{2\pi}\sum_{k\neq j}A_k\int_{-\infty}^{\infty}G_{\omega}^{(\circ)}(\bar{r}_j,\bar{r}_k)g_k(\omega)d\omega \quad , \tag{8}$$

where

$$g_k(\omega)=\int_{\circ}^{T} e^{i\omega(t-t')}S_k(t')dt' \tag{9}$$

That is, $g_k(\omega)$ is the Fourier transform of $S_k(t)$.
Also we have

$$S_j(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i\omega t}g_j(\omega)d\omega \tag{10}$$

So equation (8) is now

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i\omega t}g_j(\omega)d\omega=\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i\omega t}g_j^{(\circ)}(\omega)d\omega+\frac{1}{2\pi}\sum_{k\neq j}A_k\int_{-\infty}^{\infty} G_\omega^{(\circ)}(\bar{r}_j,\bar{r}_k)g_k(\omega)d\omega \tag{11}$$

From this equation we obtain

$$g_j(\omega)=g_j^{(\circ)}(\omega)+\sum_{k\neq j}A_kG_\omega^{(\circ)}(\bar{r}_j,\bar{r}_k)g_k(\omega) \tag{12}$$

Or in vector form

$$\bar{g}^{(\circ)}(\omega)=\bar{g}(\omega)-\underline{G}^{(\circ)}(\omega)\bar{g}(\omega) \tag{13}$$

By factorizing,

$$\bar{g}^{(\circ)}(\omega)=[\underline{1}-\underline{G}^{(\circ)}(\omega)]\bar{g}(\omega) \quad , \tag{14}$$

where

$$\underline{G}_{j,k}^{(\circ)}(\omega)=\begin{Bmatrix} 0 \quad if \quad j=k \\ A_kG_\omega^{(\circ)}(\bar{r}_j,\bar{r}_k) \quad if \quad j\neq k\end{Bmatrix} \tag{15}$$

Formally, we can invert equation (14)

$$\bar{g}(\omega)=[\underline{1}-\underline{G}^{(\circ)}(\omega)]^{-1}\bar{g}^{(\circ)}(\omega) \tag{16}$$

That is

$$\bar{g}(\omega)=\bar{g}^{(\circ)}(\omega)+\underline{G}^{(\circ)}(\omega)\bar{g}^{(\circ)}(\omega)+[\underline{G}^{(\circ)}(\omega)]^2\bar{g}^{(\circ)}(\omega)+[\underline{G}^{(\circ)}(\omega)]^3\bar{g}^{(\circ)}(\omega)+\cdots \tag{17}$$

The k component of equation (17) is

$$g_k(\omega)=g_k^{(\circ)}(\omega)+\sum_t A_kG_\omega^{(\circ)}(\bar{r}_k,\bar{r}_t)g_t^{(\circ)}(\omega)+\sum_t\sum_l A_lG_\omega^{(\circ)}(\bar{r}_k,\bar{r}_l)A_tG_\omega^{(\circ)}(\bar{r}_l,\bar{r}_t)g_t^{(\circ)}(\omega)+\cdots \tag{18}$$

Now, by substituting $g_k(\omega)$ from (18) into (12)

$$g_j(\omega) = g_j^{(\circ)}(\omega) + \sum_k A_k G_\omega^{(\circ)}(\bar{r}_j,\bar{r}_k)\{g_k^{(\circ)}(\omega) + \sum_t A_t G_\omega^{(\circ)}(\bar{r}_k,\bar{r}_t)g_t^{(\circ)}(\omega) + \sum_t \sum_l A_l G_\omega^{(\circ)}(\bar{r}_k,\bar{r}_l)A_t G_\omega^{(\circ)}(\bar{r}_l,\bar{r}_t)g_t^{(\circ)}(\omega) + \cdots\} \quad (19)$$

The brackets can be eliminated to give

$$g_j(\omega) = g_j^{(\circ)}(\omega) + \sum_k A_k G_\omega^{(\circ)}(\bar{r}_j,\bar{r}_k)g_k^{(\circ)}(\omega) + \sum_k \sum_t A_k G_\omega^{(\circ)}(\bar{r}_j,\bar{r}_k)A_t G_\omega^{(\circ)}(\bar{r}_k,\bar{r}_t)g_t^{(\circ)}(\omega) + \cdots \quad (20)$$

From this equation we obtain the discrete Neumann [3] series for the Fourier transform of the solution to the integral equation (2)

$$g_j(\omega) = g_j^{(\circ)}(\omega) + \sum_t A_t g_t^{(\circ)}(\omega)\{G_\omega^{(\circ)}(\bar{r}_j,\bar{r}_t) + \sum_k G_\omega^{(\circ)}(\bar{r}_j,\bar{r}_k)A_k G_\omega^{(\circ)}(\bar{r}_k,\bar{r}_t) + \sum_{k,l} G_\omega^{(\circ)}(\bar{r}_j,\bar{r}_k)A_k G_\omega^{(\circ)}(\bar{r}_k,\bar{r}_l)A_l G_\omega^{(\circ)}(\bar{r}_l,\bar{r}_t) + \cdots\} \quad (21)$$

If the expression between brackets in (21) converges, it must be equal to the Fourier transform of the complete Green's function $G_\omega(\bar{r}_j,\bar{r}_t)$, so that we can write

$$g_j(\omega) = g_j^{(\circ)}(\omega) + \sum_k A_k G_\omega(\bar{r}_j,\bar{r}_k)g_k^{(\circ)}(\omega) \quad (22)$$

Equation (22) can be written in vector form as

$$\bar{g}(\omega) = [\underline{1} + \underline{G}(\omega)]\bar{g}^{(\circ)}(\omega) \quad , \quad (23)$$

where

$$\underline{G}(\omega)_{i,j} = \begin{Bmatrix} 0 \quad if \quad j=k \\ A_j G_\omega(\bar{r}_i,\bar{r}_j) \quad if \quad j \neq k \end{Bmatrix} \quad (24)$$

Equations (23) and (24) constitute our basic tools to perform the time inversion, that is, we can use equation (23) to obtain experimental data of the components $G_\omega(\bar{r}_j,\bar{r}_k)$, because we know the Fourier transforms of the original signals $g_k^{(\circ)}(\omega)$, and we can measure the arriving signals $g_j(\omega)$.

4. THE TIME INVERSION OF A SIGNAL

Let us carry out a time inversion. Suppose we have transmitted a signal burst from the site $\bar{r}_s$ and that we have recorded the arrived signals during a time T by the set $\{\bar{r}_b\}$. We can reverse the signal from each of the $\{\bar{r}_b\}$; for example, the signal $u_j(t)$ recorded by $\bar{r}_j$ can be emitted in reverse order[1] to obtain a contribution at the original place as

$$u_s^{(\circ)}(t) = \int_{-\infty}^{\infty} G^*(\bar{r}_j, T-t'; \bar{r}_s, t)u_j(T-t')dt' \quad (25)$$

Or in terms of $S_j(t)$

$$S_j^{(\circ)}(t) = \int_{\circ}^{T} G^*(\bar{r}_j, T-t'; \bar{r}_s, t)S_j(T-t')dt' \quad (26)$$

We can express (26) in terms of the Fourier transform $G^*_\omega(\bar{r}_j, \bar{r}_s)$:

$$G^*(\bar{r}_j, T - t'; \bar{r}_s, t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} G^*_\omega(\bar{r}_j, \bar{r}_s) e^{-i\omega(T-t'-t)} d\omega \tag{27}$$

That is

$$S_s^{(\circ)}(t) = \int_\circ^{\mathrm{T}} \frac{1}{2\pi}\int_{-\infty}^{\infty} G^*_\omega(\bar{r}_j, \bar{r}_s) \times e^{-i\omega(\mathrm{T}-t'-t)} d\omega S_j(\mathrm{T}-t')dt' \quad , \tag{28}$$

which can be rearranged as

$$S_s^{(\circ)}(t) = \int_{-\infty}^{\infty} \frac{1}{2\pi} G^*_\omega(\bar{r}_j, \bar{r}_s) e^{-i\omega(\mathrm{T}-t)} \times [\int_\circ^{\mathrm{T}} e^{i\omega t'} S_j(\mathrm{T}-t')dt']d\omega \tag{29}$$

Based on properties of the Fourier transform, equation (29) is equivalent to

$$S_s^{(\circ)}(t) = \int_{-\infty}^{\infty} \frac{1}{2\pi} G^*_\omega(\bar{r}_j, \bar{r}_s) e^{-i\omega(\mathrm{T}-t)} \times e^{i\omega\mathrm{T}}[\int_\circ^{\mathrm{T}} e^{-i\omega\upsilon} S_j(\upsilon)d\upsilon]d\omega \tag{30}$$

Or

$$S_s^{(\circ)}(t) = \int_{-\infty}^{\infty} \frac{1}{2\pi} G^*_\omega(\bar{r}_j, \bar{r}_s) e^{i\omega t} g_j(\omega) d\omega \tag{31}$$

And if we also put the Fourier development of $S_s^{(\circ)}(t)$:

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i\omega t} g_s^{(\circ)}(\omega) d\omega = \int_{-\infty}^{\infty} \frac{1}{2\pi} G^*_\omega(\bar{r}_j, \bar{r}_s) e^{i\omega t} g_j(\omega) d\omega \tag{32}$$

We get

$$g_s^{(\circ)}(\omega) = G^*_\omega(\bar{r}_j, \bar{r}_s) g_j(\omega) \quad , \tag{33}$$

where $g_j(\omega)$ is the Fourier transform of the signal received at $\bar{r}_j$, while $g_s^{(\circ)}(\omega)$ is the Fourier transform of the signal that returned to $\bar{r}_s$ after it has been reflected.

If we want the total contributions at $\bar{r}_s$ from the set of transducers $\{\bar{r}_j\}$ we must do the sum

$$S_s^{(\circ)}(t) = \sum_j \int_\circ^{\mathrm{T}} G^*(\bar{r}_j, \mathrm{T}-t'; \bar{r}_s, t) S_j(\mathrm{T}-t')dt' \quad , \tag{34}$$

which gives

$$g_s^{(\circ)}(\omega)=\sum_j G_\omega^*(\bar{r}_j,\bar{r}_s)g_j(\omega) \tag{35}$$

5. SENDING A MESSAGE TOWARDS A SITE.

Suppose now that our objective is to send a message from $\bar{r}_j$, that arrives at $\bar{r}_s$. Then we write [4]

$$u_s^{(\circ)}(t)=\int_{-\infty}^{\infty} G^*(\bar{r}_j,\mathrm{T}-t';\bar{r}_s,t)u_j(t')dt' \tag{36}$$

From this and equation (27), we have

$$S_s^{(\circ)}(t)=\int_{-\infty}^{\infty}\frac{1}{2\pi}G_\omega^*(\bar{r}_j,\bar{r}_s)e^{-i\omega(\mathrm{T}-t)}\times[\int_\circ^{\mathrm{T}} e^{i\omega t'}S_j(t')dt']d\omega \tag{37}$$

This can be rewritten as

$$S_s^{(\circ)}(t)=\int_{-\infty}^{\infty}\frac{1}{2\pi}G_\omega^*(\bar{r}_j,\bar{r}_s)e^{-i\omega(\mathrm{T}-t)}\times[\int_\circ^{\mathrm{T}} e^{-i\omega t'}S_j^*(t')dt']^*d\omega \tag{38}$$

And by expressing $S_{\cdot s}^{(\circ)}(t)$ and $S_j(t')$ in terms of their Fourier series development we obtain

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}e^{i\omega t}g_s^{(\circ)}(\omega)d\omega=\int_{-\infty}^{\infty}\frac{e^{-i\omega\mathrm{T}}}{2\pi}G_\omega^*(\bar{r}_j,\bar{r}_s)e^{i\omega t}g_j^*(\omega)d\omega \tag{39}$$

This gives

$$g_s^{(\circ)}(\omega)=e^{-i\omega\mathrm{T}}G_\omega^*(\bar{r}_j,\bar{r}_s)g_j^*(\omega)\quad , \tag{40}$$

where $g_j^*(\omega)$ is the complex conjugate of the Fourier transform of the message sent from $\bar{r}_j$, while $g_s^{(\circ)}(\omega)$ is the Fourier transform of the signal that was selected to arrive at $\bar{r}_s$.

For a batch of different messages from $\bar{r}_j$ that go to the precise sites $\bar{r}_{s_1}, \bar{r}_{s_2}, \bar{r}_{s_3}, \ldots, \bar{r}_{s_n}$, we have the sum

$$\sum_q g_{s_q}^{(\circ)}(\omega)=\sum_q e^{-i\omega\mathrm{T}}G_\omega^*(\bar{r}_j,\bar{r}_{s_q})g_{j\to q}^*(\omega)\quad , \tag{41}$$

where $j\to q$ means that the signal begins at $\bar{r}_j$ and its destination is $\bar{r}_{s_q}$.

6. A SURVEY OF APPLICATIONS OF TIME INVERSION

As we said in the introduction, we developed a tool that can be applied to a broad class of phenomena where time reversal is involved, and we gave the mathematical support. Let us recall several specific applications such as those mentioned at the introduction. For the development of a device for destroying brain tumors or kidney stones [5] the first step would be the localization of the abnormal sites with the help of an explorer emission and by means of the above described inversion techniques which show and record the interesting points using a set of transducers. Then the recorded signal is greatly amplified, sent, and returned backward in time toward the tumor or the stone to destroy it. To improve the results we can repeat the exploration procedure several times before the final amplification. Regarding the improvement of sonar underwater [5] detection, the procedure would basically consist again in the emission of an exploration signal that is received and recorded tens of kilometers away from an initial sound source by a series of transducers known generally as a Time Reverse Mirror (TRM); once the signal has been returned by the TRM, another collection of transducers near the initial signal source record the remaining returned signal, resulting in a very localized area in a very localized interval of time, and thus the TRM has been calibrated by the original signal and any new obstacle can be detected easily. Another objective is to learn how to implement the process of sending a message from one location to another very specific location. So, our immediate work is to implement an experimental measure of the Green's functions for simple geometries and simple arrangements of transducers. For instance we can seek to improve the focus of a TRM reflected signal by increasing both the number of obstacles in the signal trajectories and the time T during which the incident signal is recorded, phenomena that are explained by an increment of the cross section of the receptors as the signal collides with each obstacle, thus improving the signal definition, as was reported by other authors [4].

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Authors Biography

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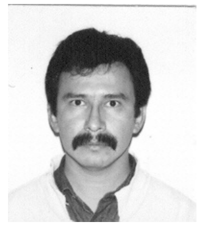

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