

ONE SIMPLE METHOD FOR THE DESIGN OF MULTIPLIERLESS FIR FILTERS

G. Jovanovic-Dolecek¹, M. M-Alvarez² & M. Martinez³

¹INAOE, Puebla, México
gordana@inaoep.mx

²IT, Mérida, México

³Universidad Politecnica de Valencia, Spain

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ABSTRACT

This paper presents one simple method for the design of multiplierless finite impulse response (FIR) filters by the repeated use of the same filter. The prototype filter is a cascade of a second order recursive running sum (RRS) filter, known as a cosine filter, and its corresponding expanded versions. As a result, no multipliers are required to implement this filter.

RESUMEN

Este artículo presenta un método simple para el diseño de filtros digitales con respuesta al impulso finita (FIR) sin multiplicadores, usando el mismo filtro varias veces. El filtro prototipo es una cascada de los filtros recursivos corrientes (RRS) de segundo orden conocidos como filtros cosenos. Así mismo se usan versiones expandidas de estos filtros. Como resultado no se necesitan ningunos multiplicadores para implementar este filtro.

KEYWORDS: FIR Filters, Multiplierless Filters, Sharpening, Cosine Filters.

1. INTRODUCTION

Digital signal processing (DSP) is an area of science and engineering that has been rapidly developed over the past years [1 & 2]. This rapid development is the result of significant advances in digital computers technology and integrated circuits fabrication [3-5].

A typical operation in DSP is filtering. The function of a digital filter is to process a given input sequence $x(n)$ and generate the output sequence $y(n)$ with the desired characteristics. If the input signal $x(n)$ is the *unit sample sequence*, the output signal would be the characteristic of the filter, called the *unit sample response* $h(n)$.

Depending on the length of $h(n)$, the filters can be either FIR (*Finite impulse response filters*), or IIR (*Infinite impulse response filters*).

FIR filters are often preferred over IIR filters because they have several very desirable properties such as linear phase, stability, and absence of limit cycle. The main disadvantage of FIR filters is that they involve a higher degree of computational complexity compared to IIR filters with equivalent magnitude response [6-8].

FIR filters of length N require $(N+1)/2$ multipliers if N is odd, $N/2$ multipliers if N is even, and $N-1$ adders and $N-1$ delays. The complexity of the implementation increases with the increase in the number of multipliers.

Over the past years there has been a number of attempts to reduce the number of multipliers [9-12] etc.

Another approach is a true multiplier-free (also referred as multiplierless) design where the coefficients are reduced to simple integers or to simple combinations of powers of two, for example [13-16], etc.

One alternative approach is based on combining simple sub-filters which do not require any multipliers. Ramakrishnan and Gopinathan [11] proposed to sharpen a cascade of *recursive running sum* (RRS) filters which are multiplier-free filters.

Tai and Lin [13] proposed a design of multiplier-free filters based on sharpening technique where the prototype filter is a cascade of the cosine filters which requires no multipliers and only some adders. However, to satisfy the desired specification the order of the sharpening polynomial must be high, thereby resulting in high complexity.

In this paper we propose a modification of this method which results in lower overall complexity. The paper is organized in the following way. In Section 2 we describe cosine filters, while in Section 3 we present the sharpening technique. The proposed design, along with one example is given in Section 4.

2. COSINE FILTERS

The simplest low pass finite impulse response (FIR) filter is the M -point *moving-average* (MA) filter, also known as the *comb filter*, with an impulse response

$$g(n) = \begin{cases} 1/M, & \text{for } 0 \leq n \leq M-1 \\ 0, & \text{otherwise,} \end{cases} \quad (1)$$

where M is an integer. Its system function is given by

$$G(z) = \frac{1}{M} (1 + z^{-1} + z^{-2} + \dots + z^{-(M-1)}) = \frac{1}{M} \sum_{k=0}^{M-1} z^{-k} \quad (2)$$

The scaling factor $1/M$ is needed to provide a dc gain of 0 dB. This filter does not require any multiplications or coefficient storages. A more convenient form of the previous system function for realization purposes is given by

$$G(z) = \frac{1}{M} \left(\frac{1 - z^{-M}}{1 - z^{-1}} \right), \quad (3)$$

which is also known as a *recursive running-sum filter* (RRS) [9].

For example, for $M=2$, we have

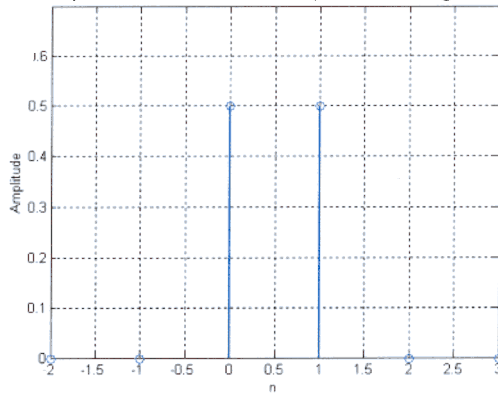
$$G(z) = \frac{1}{2} \left(\frac{1 - z^{-2}}{1 - z^{-1}} \right) = \frac{1}{2} (1 + z^{-1}) \quad (4)$$

The magnitude response of the filter for $M=2$ is

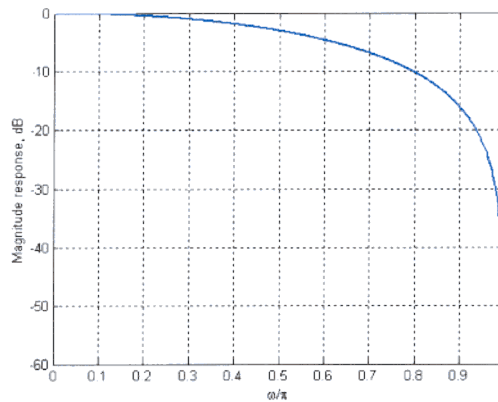
$$|G(e^{j\omega})| = |\cos(\omega/2)| \quad (5)$$

Because of this cosine form this filter is called a *cosine filter*.

The impulse response and magnitude response of this filter are plotted in Figure 1



a) Impulse response



b) Magnitude response

Figure 1. Cosine filter

The N -expanded filter is obtained by inserting $N-1$ zeros between each sample of the impulse response. In z-domain that means that each delay is replaced by N delays

$$G(z^N) = \frac{1}{2}(1 + z^{-N}). \quad (6)$$

The corresponding magnitude response is

$$|G(e^{j\omega N})| = |\cos(N\omega/2)|. \quad (7)$$

Figure 2 shows the impulse responses and the magnitude responses of the expanded cosine filters for different values of N .

The sequences in Figure 2 are expanded in time domain by a factor of N , and in the frequency domain are compressed by the same factor N . As a consequence, the frequency of the first zero of the magnitude response at π in Figure 1b would be compressed by N . For example for $N=2$ it would be at $\pi/2$, (Fig. 2a), for $N=3$ at $\pi/3$, (Figure 2b), for $N=4$ at $\pi/4$, (Figure 2c), etc.

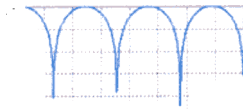
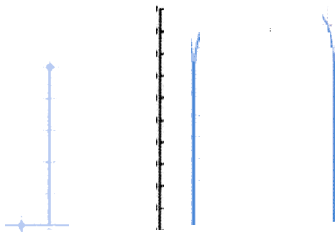
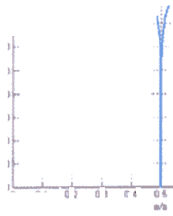


Fig 2

100

10000

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We can also notice that the cascade of different expanded cosine filters will result in one low pass magnitude characteristic because the zeros of one filter will cancel the side lobes of the adjacent filter.

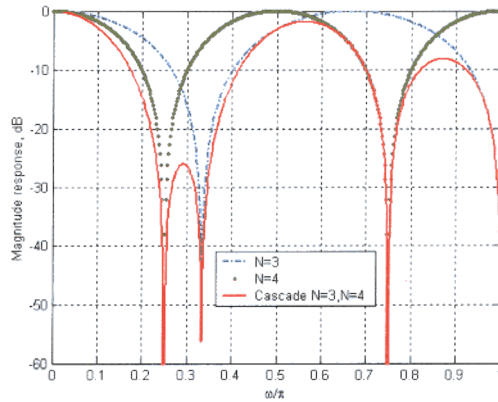


Figure 3. Two expanded cosine filters and their cascade

As illustrated in Figure 3, the zero of the filter in Figure 2c at frequency $\omega = 0.75 \pi$ decreases the side lobe of the filter in Figure 2b around that frequency.

The transfer function of the cascade of K expanded cosine filters is given by

$$H(z) = \prod_{N=1}^K G(z^N) = \prod_{N=1}^K \frac{1}{2} (1 + z^{-N}) \quad (8)$$

The corresponding magnitude response is then

$$\left| H(e^{j\omega}) \right| = \left| \prod_{N=1}^K G(e^{j\omega N}) \right| = \left| \prod_{N=1}^K \cos(N\omega/2) \right| \quad (9)$$

In Figure 4, magnitude responses (9) are plotted for different values of K .

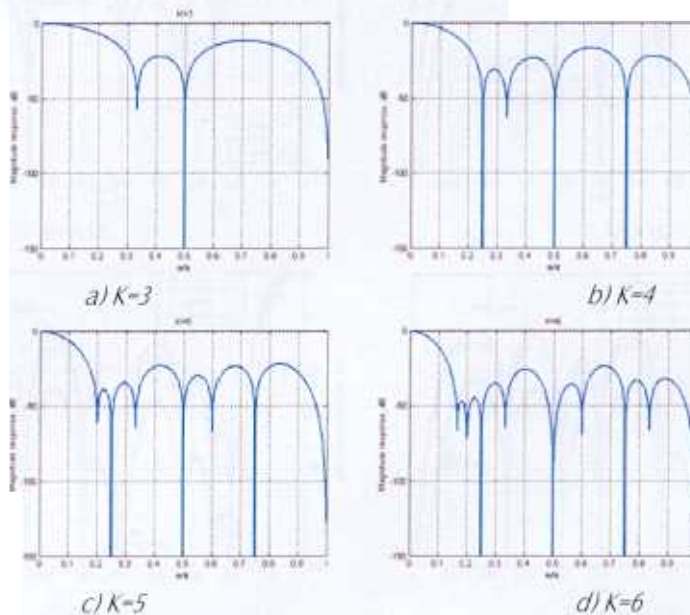


Figure 4. Magnitude response for the cascade of K cosine filters

We can notice that

The cascade of cosine filters is multiplier-free. (See Equation 8).

- The magnitude characteristic is low pass but it has a bad stop band attenuation and significant pass band droop.

In order to improve the magnitude characteristic of this cascade, we use the sharpening technique, as explained in the following section.

3. SHARPENING TECHNIQUE

The filter sharpening technique introduced by Kaiser and Hamming, [17], can be used for simultaneous improvement of both pass band and stop band characteristics of a linear-phase FIR digital filter. The technique uses amplitude change function (ACF). An ACF is a polynomial relationship of the form $H_0=f(H)$ between the amplitudes of the overall and the prototype filters, H_0 and H , respectively. The improvement in the pass band, near $H=1$, or in the stop band, near $H=0$, depends on the order of tangencies m and n of the ACF at $H=1$ or at $H=0$.

The expressions for ACF, proposed by Kaiser and Hamming [17], for the m^{th} and n^{th} order tangencies of the ACF at $H=1$ and $H=0$, respectively, are given as,

$$H_0 = H^{n+1} \sum_{k=0}^m \frac{(n+k)!}{n!k!} (1-H)^k = H^{n+1} \sum_{k=0}^m C(n+k, k) (1-H)^k \quad (10)$$

where $C(n+k, k)$ is the binomial coefficient.

The values of the ACF for some typical values of m and n are given in Table

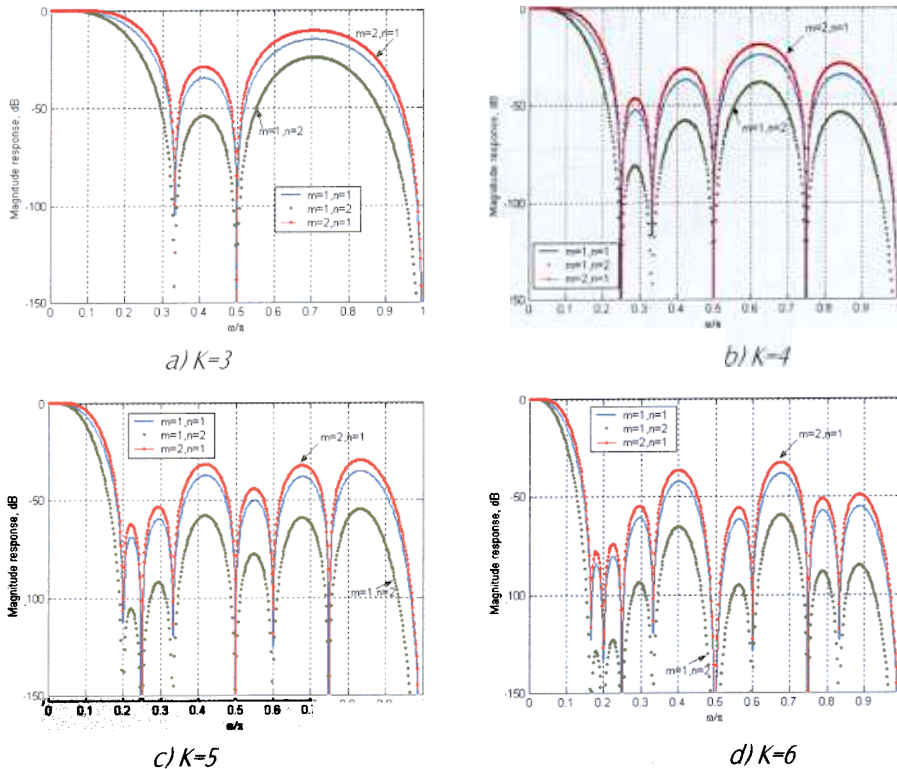


Figure 5. Sharpening of the cascade of cosine filters

Figure 5 shows the results of applying the sharpening technique to cosine filters of Figure 4. Three different ACF's from Table I are used: $m = n = 1$; $m = 1, n = 2$; and $m = 2, n = 1$. Notice that the pass band is improved by increasing m , while the stop band is improved by increasing n .

In order to obtain more flexibility in the design, instead of using a single filter $H(z)$ in sharpening polynomials of the Table I, we can use a cascade of k filters $H(z)$. For example, the sharpening polynomial for $m=1, n=1$ and $k=1$ is $3H^2 - 2H^3$. If we use $k=2$ filters H in the cascade, the corresponding polynomial is $3H^4 - 2H^6$.

TABLE I. ACF polynomials for $m = 1, 2, 3$ and $n=1, 2, 3$

m	n	ACF polynomials
1	0	$2H - H^2$
1	1	$3H^2 - 2H^3$
1	2	$4H^3 - 3H^4$
1	3	$5H^4 - 4H^5$
1	4	$6H^5 - 5H^6$
2	0	$H^3 - 3H^2 + 3H$
2	1	$3H^4 - 8H^3 + 6H^2$
2	2	$6H^5 - 15H^4 + 10H^3$
2	3	$10H^6 - 24H^5 + 15H^4$
2	4	$15H^7 - 35H^6 + 21H^5$
3	0	$-H^4 + 4H^3 - 6H^2 + 4H$
3	1	$-4H^5 + 15H^4 - 20H^3 + 10H^2$
3	2	$-10H^6 + 36H^5 - 45H^4 + 20H^3$
3	3	$-20H^7 + 70H^6 - 84H^5 + 35H^4$

Figure 6a compares the results for the cascade of $K=3$ cosine filters (8) using the polynomial with $m=1, n=1, k=2$, polynomial with $m=1, n=3, k=1$, and the polynomial with $m=1, n=4, k=1$.

The corresponding pass bands are plotted in Figure 6b

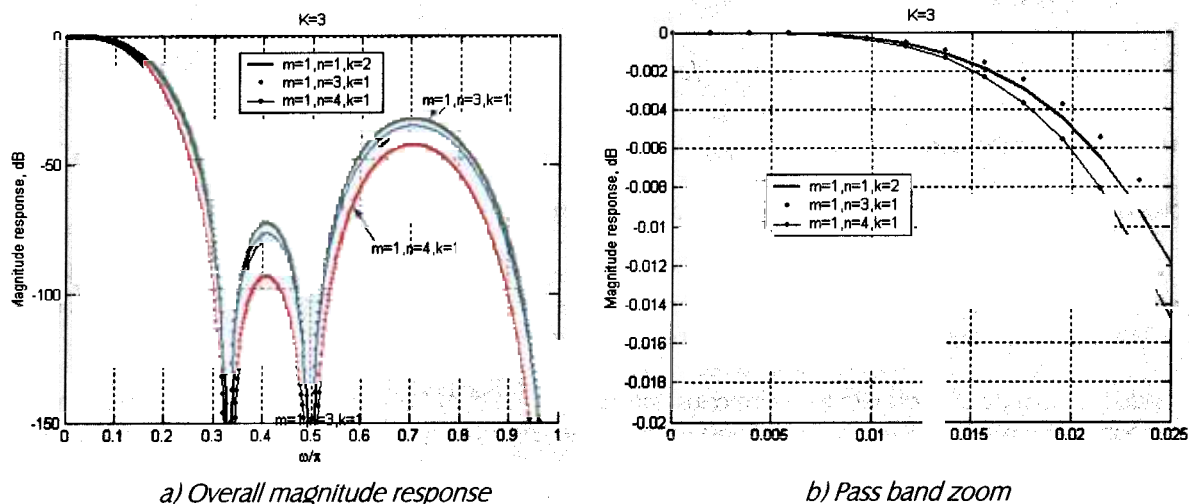


Figure 6. Sharpening of the cosine filters, $K=3$

We can notice that when the parameter $k=2$ in the sharpening polynomial $m=1$ and $n=1$ is used, a new polynomial which fills the characteristics between the polynomial $m=1, n=3$, and the polynomial $m=1, n=4$ results.

Similarly, using different values of k in the polynomials of Table we obtain new polynomials which fit the characteristics of the original polynomials of the Table I.

Figure 7 demonstrates polynomials with $K=6, m=1, n=1$, and $k=1, 2$, and 3

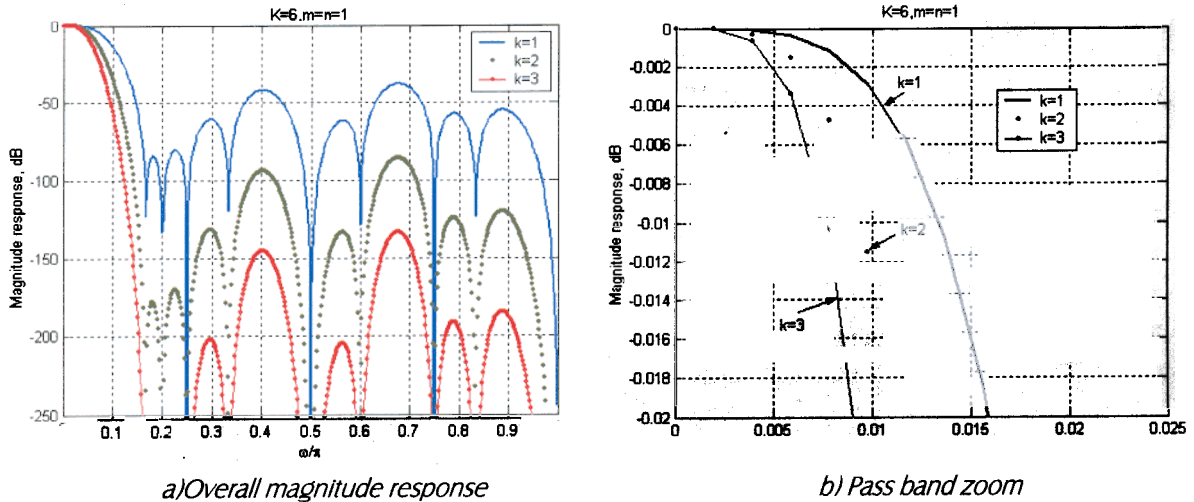


Figure 7. Sharpening using a cascade of cosine filters, $K=6$

As explained in the next section, we can also combine different ACF's for a better improvement in both the pass band and the stop band.

4. DESIGN PROCEDURE

We consider a typical low pass filter with the pass band edge ω_p and the stop band edge ω_s . The stop band frequency determines the number K as follows [13]

$$K = \text{int} \left\{ 1/\omega_s \right\} \quad (11)$$

where $\text{int}\{X\}$ means the closest integer of X .

We propose to apply different ACF's to different groups of filters of the cascade (8).

The filters in the cascade (8) with higher value N have more droop in the pass band and more side lobes, and therefore require an ACF with higher values of m and n . At the other extreme, the filters with smaller values of N have wider pass band and fewer side lobes and consequently require lower values of m and n in the ACF.

We divide the cascade of filters into the subgroups and apply different ACF to each subgroup. The number of filters in a subgroup is at most 3. As a result, the highest values for m and n are typically 3. The procedure is implemented in MATLAB.

The following example illustrates the method.

Example 1:

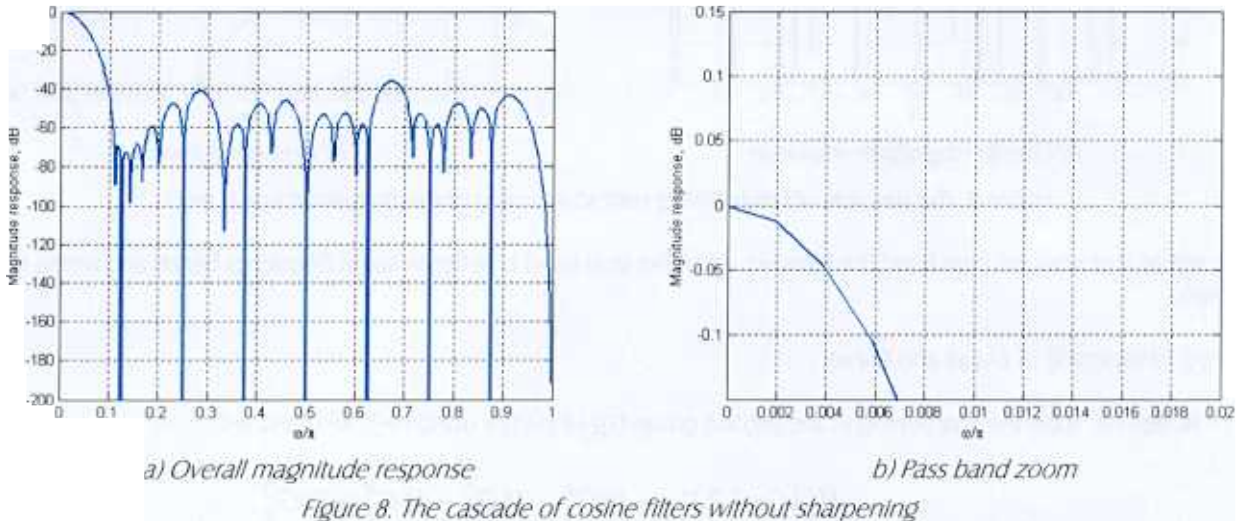
We design a filter with the following specifications: The pass band and stop band frequencies are $\omega_p=0.2$ and $\omega_s=1.1$, respectively. The pass band ripple is $R_p=0.15$ dB and the stop band attenuation is $A_s = -80$ dB. A design using the Parks McClellan algorithm results in a filter of order 78 and requires 39 multipliers.

From (11) it follows that $K=9$.

The prototype filter has the magnitude response given by

$$|H(e^{j\omega})| = \cos(\omega/2) \cos(\omega) \cos(3\omega/2) \dots \cos(9\omega/2)$$

This magnitude response and the pass band zoom are shown in Figure 8.



We can notice that both the pass band and the stop band specification are not satisfied. To remedy this problem, we first form the following groups of cosine filters:

$$G_1(\omega) = \cos(9\omega/2) \cos(8\omega/2) \cos(7\omega/2), \quad G_2(\omega) = \cos(5\omega/2) \cos(6\omega/2)$$

$$G_3(\omega) = \cos(4\omega/2) \cos(3\omega/2), \quad G_4(\omega) = \cos(\omega), \quad G_5(\omega) = \cos(\omega/2).$$

In the following steps we apply various sharpening polynomials to groups in (13).

Step 1: Sharpening of $G_1(\omega)$

We apply the ACF function with $m=3$, $n=3$, and $k=1$ to $G_1(\omega)$,

$$H_1(G_1, 3, 3, 1) = 35G_1^4 - 84G_1^5 + 70G_1^6 - 20G_1^7$$

The corresponding magnitude response is

$$|H_1(\omega)| = |F\{H_1(G_1, 3, 3, 1)\}G_2(\omega)G_3(\omega)G_4(\omega)G_5(\omega)|$$

where $\mathcal{F}\{H\}$ means Fourier transform of H .

Figure 9 demonstrates the overall and the pass band magnitude response (15).

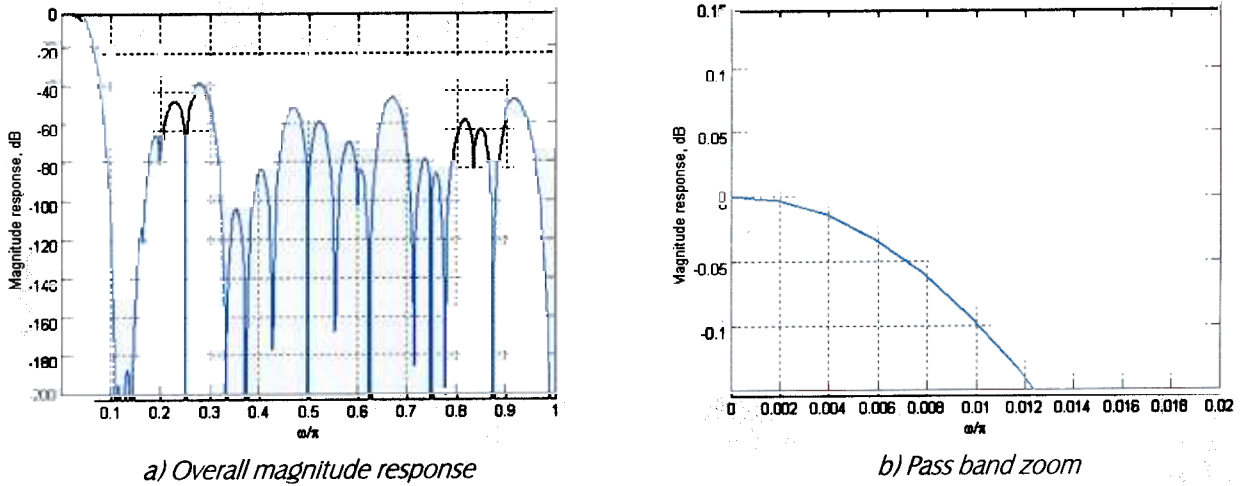


Figure 9. The cascade of cosine filters with sharpening of the first group ($m=3$; $n=3$)

We notice that now the pass band is improved, while the stop band is better in some frequency bands and worse in others.

Step 2: Sharpening of $G_1(\omega)$ and $G_2(\omega)$

In this step we apply the sharpening to the second group $G_2(\omega)$ in (13), using $m=3$, $n=2$, and $k=$

$$H_2(G_2,3,2,1) = -10G_2^6 + 36G_2^5 - 45G_2^4 + 20G_2^3$$

The corresponding magnitude response is

$$|H_{12}(\omega)| = |F\{H_1(G_1,3,3,1)\}F\{H_2(G_2,3,2,1)\}G_3(\omega)G_4(\omega)G_5(\omega)|$$

The response of this cascade is shown in Figure 10.

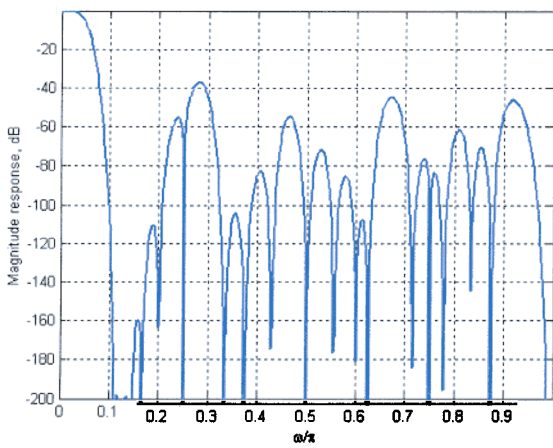
A similar procedure is outlined in the remaining steps.

Step 3: Sharpening of $G_1(\omega)$, $G_2(\omega)$ and $G_3(\omega)$

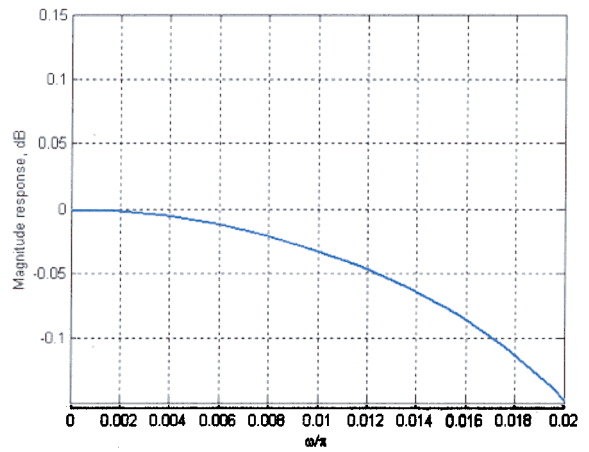
$$H_3(G_3,2,2,1) = 6G_3^5 - 15G_3^4 + 10G_3^3$$

$$|H_{123}(\omega)| = |F\{H_1(G_1,3,3,1)\}F\{H_2(G_2,3,2,1)\} \times F\{H_3(G_3,2,2,1)\}G_4(\omega)G_5(\omega)| \quad (19)$$

The result is shown in Figure 11



a. Overall magnitude response



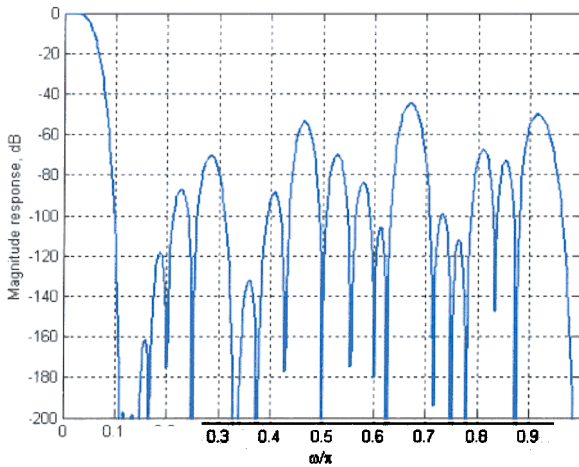
b. Pass band zoom

Figure 10. The cascade of cosine filters with sharpening of the first ($m=3$; $n=3$) and the second group ($m=3$; $n=2$)

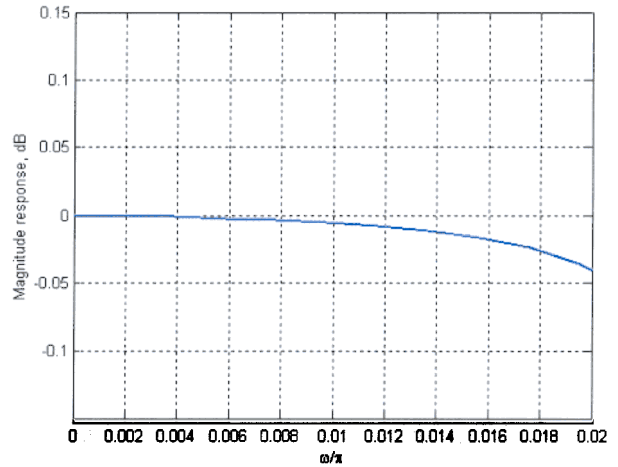
Step 4: Sharpening of $G_1(\omega)$, $G_2(\omega)$, $G_3(\omega)$ and $G_4(\omega)$

$$H_4(G_4,0,3,1) = G_4^4$$

$$H_{1234}(\omega) = F\{H_1(G_1,3,3,1)\}F\{H_2(G_2,3,2,1)\}F\{H_3(G_3,2,2,1)\} \times F\{H_4(G_4,0,3,1)\}G_5(\omega) \quad (21)$$



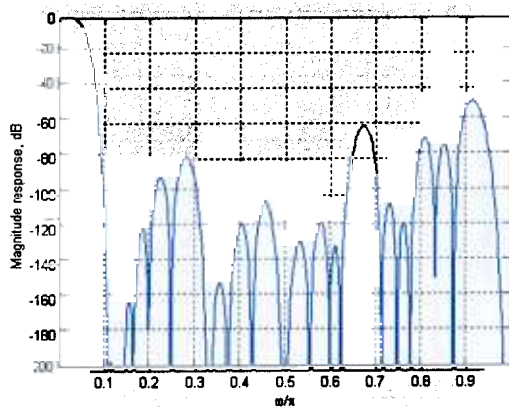
a. Overall magnitude response



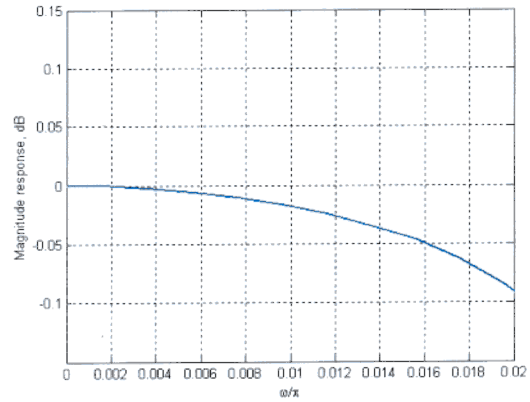
b. Pass band zoom

Figure 11. The cascade of cosine filters with sharpening of the first ($m=3$; $n=3$), second ($m=3$; $n=2$), and third group ($m=2$; $n=2$)

The result is illustrated in Figure 12.



a. Overall magnitude response



b. Pass band zoom

Figure 12. The cascade of cosine filters with sharpening of the first ($m=3; n=3$), second ($m=3; n=2$), third ($m=2; n=2$), and the fourth group ($m=0; n=3$)

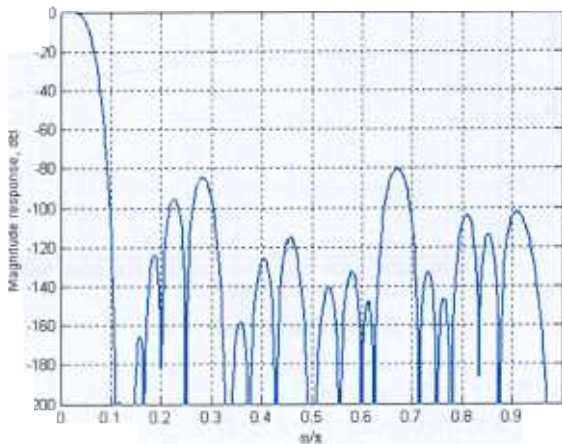
Step 5: (Last step). Sharpening of $G_1(\omega)$, $G_2(\omega)$, $G_3(\omega)$, $G_4(\omega)$ and $G_5(\omega)$

$$H_5(G_4, 0, 3, 1) = G_5^4 \quad (22)$$

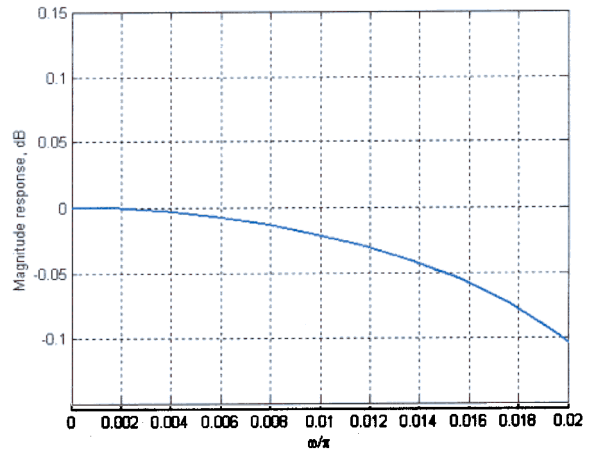
$$H_{12345}(\omega) = |F\{H_1(G_1, 3, 3, 1)\}F\{H_2(G_2, 3, 2, 1)\}F\{H_3(G_3, 2, 2, 1)\} \times F\{H_4(G_4, 0, 3, 1)\}F\{H_5(G_5, 0, 3, 1)\}| \quad (23)$$

Figure 13 shows the magnitude responses of the designed filter. We can notice that both the pass band and stop band specifications are satisfied.

In order to compare the proposed result with the method of Tai and Lin [13], we apply the polynomial $m=2, n=2, k=1$ to all cascades. The result in Figures 14a and 14b shows that the specification is not satisfied.

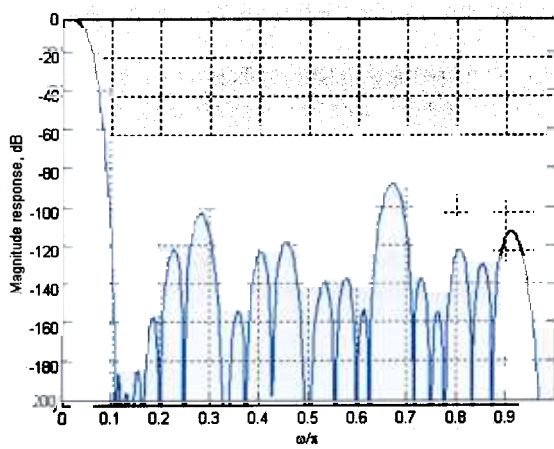


a. Overall magnitude response

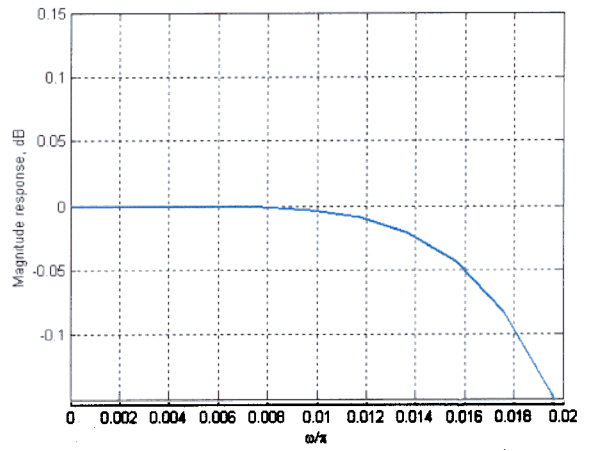


b. Pass band zoom

Figure 13. The designed filter



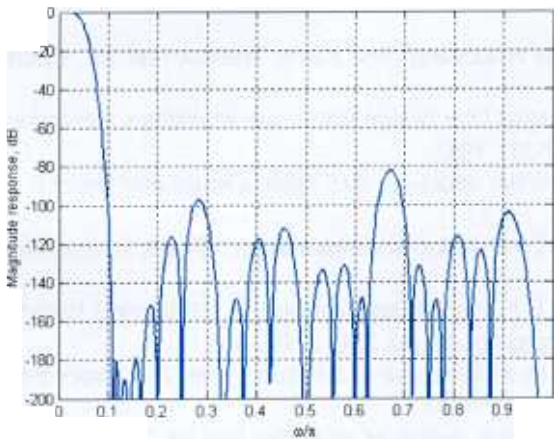
a) Overall magnitude response



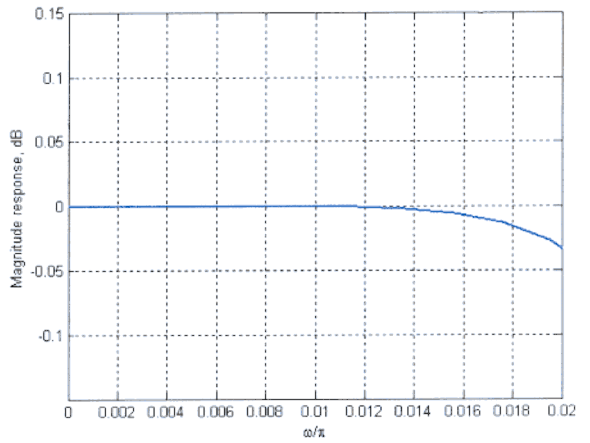
b) Pass band zoom

Figure 14. Method of Tai and Lin [13] ($m=n=2$)

However, the polynomial with $m=3$, $n=2$, $k=1$, satisfies the specification (Figure 15). Therefore, this example shows that the proposed method results in a less complex filter than the one proposed by Tai and Lin.



a. Overall magnitude response



b. Pass band zoom

Figure 15. Method of Tai and Lin [13] ($m=3$, $n=2$).

5. CONCLUSIONS

A simple method for the design of multiplierless FIR filters is presented. The method uses a cascade of second order RRS filters with the corresponding expanded filters. The number of filters in the cascade depends on the value of the stop band frequency. In contrast to the method proposed in Tai and Lin, 1992, where the orders of tangencies n and m are varied from 1 to 8, in the method proposed here the orders of tangencies are varied only from 1 to 3, thereby resulting in a less complex filter.

A less complex design is achieved by dividing up the cascade of K cosine filters into the subgroups. The number of filters in a subgroup typically varies from 1 to 3. The designed examples show that it is a good choice to start with a

subgroup consisting of 3 elements for higher values of N in (8). The size of the subgroups is decreased for lower values of N , and for $N=1$ it is typically 1.

In order to permit more flexibility in the design, the prototype filter in sharpening polynomials is a cascade of k filters (8). The complexity of the sharpening polynomials decreases with the decrease in the corresponding values of N .

If the designed filter satisfies the given specification, in the following iteration one could try to decrease complexity by decreasing k and the corresponding m and n values of the sharpening polynomials.

The proposed method is intended for the narrow band FIR filter design. As a future work, it would be interesting to extend the procedure to also include wide band FIR filters as well as to analyze its implementation issues.

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