# MEASUREMENT OF CHUA CHAOS AND ITS APPLICATIONS

Ricardo Núñez Pérez

Electronics and Telecommunications Department
Center for Scientific Research and Graduate Studies of Ensenada-CICESE
Km.107, Tijuana.-Ensenada Highway, Ensenada, B.C., México, 22860.
Tel/Fax (646)175-0555/554
rnunez@cicese.mx

ABSTRACT

The present paper shows a single and original measurement circuit, using instantaneous and average readouts of the chaotic dynamic of a Chua's circuit $Vc_1(x_1)$ signal with binary decryption, synchrony qualification and chaotic sequence applications. The circuit was simulated (Workbench), developed and tested with our Dadisp Test Automatic System. In operation, the circuit was efficient and successful and could be redesigned to quantify the chaotic dynamic of multiple scrolls and time-delay Chua circuits.

KEYWORDS: Chaotic dynamics, Chua's circuit, chaotic switching, binary decryption, chaotic circuits synchronization.

RESUMEN

Se presenta un circuito original para medir la dinámica caótica del circuito de Chua (señal Vc1(x1)) en forma instantánea y promedio. Se aplica en el desencriptamiento binario, la calificación de la sincronización de circuitos y en la generación de secuencias. El circuito, se simula (Workbench) y se desarrolla y prueba con nuestro Sistema Automático de Prueba Dadisp. Los resultados son aceptables y se está rediseñando para considerar dinámicas caóticas de circuitos con enrollamientos múltiples y con retardo de tiempo.

PALABRAS CLAVE: Dinámica caótica, circuito de Chua, conmutación caótica, desencriptamiento y sincronización caótica

## 1. INTRODUCTION

The idea for the original project [1] came from a study performed to experimentally classify the synchronization of two chaotic circuits [2, 3] in terms of wave forms, spectrums, phase planes and analysis of coherence when parameters vary and noise is introduced into the synchronizing signal. The analysis of coherency [2] produces the best results when working with time and frequency. But nevertheless, if we wish to quantify the chaotic dynamics of the $Vc_1(x_1)$ of the Chua circuit [4], it is simpler to start with the waveform and its phase plane (attractor) since, as we already know, a direct relationship between these and the equilibrium points exists [4, 5]. In the second section, the Chua circuit is described; and in the one that follows, it *quantifies the chaotic dynamics of the $Vc_l(x_1)$ signal of the circuit in question* with the help of the chaos meter, -based on the Workbench program (Wb)-, and the following procedure The procedure consists of instantly counting the number of times this signal crosses the equilibrium levels during a cycle called the *"chaotic cycle".* If an average is calculated during several of these cycles, a value representative of this dynamic can be found. In the fourth section, four applications of the Chua chaos meter are described: *a)* Instantaneous *counting* of spins/turns per chaotic cycle *b) Decryption* of binary information sent through one line, -through parametrical commutation [6-10] during several chaotic cycles-, *c) Qualifying* of the Chua circuit synchronization [11- 13] per chaotic cycle, based on ascertaining the numerical difference between counted events for the $Vc_1(x_1)$

synchronizer signal and the $Vc_l$~($x_1$~) synchronized signal, and *d) Generation* of chaotic test sequences. Finally, acknowledgments, conclusions, and planned future projects are presented (e.g., to condition the circuit to count Chua circuit dynamics with multiple scrolls and time-delay) and the most relevant references.

## 2. THE CHUA CIRCUIT

The Chua circuit [4] is known as double-scroll, because it produces an attractor in which its dynamics passes from one scroll to another. This circuit is autonomous and non-linear, and it is comprised of two capacitors, a coil, two resistors, and a non-linear element. This latter is a linear conductance in segments which is defined by the function $f(x_1)$ (2). The dynamics of the circuit is described by the normalized integral-differential equations (1):

$$\begin{aligned} dtx_1 &= \alpha( x_2 - x_1 - f(x_1)), \\ dtx_2 &= x_1 - x_2 + x_3, \\ dtx_3 &= -\beta x_2, \end{aligned} \qquad (1)$$

in which: $x_2$, $x_1$ and $x_3$ are the variables or dynamic signals which represent the voltage through $C_2$ , $C_1$, and the current in L, respectively. The lack of linearity in the generation of chaotic behavior in the circuit is ensured with the $f(x_1)$ function (2), called the Chua diode [4, 5], whose phase plane is presented in figure 1. Said plane shows the rupture points and the slope changes are shown in complete characterization obtained in the say described by Kennedy [4]. Although, for the circuit studied, only $m_0$ and $m_1$ slopes and their two rupture points are shown. Said function is defined as:

$$f(x_1)= bx_1 + 0.5(a - b)[| x_1 + 1| – | x_1 - 1|] , \qquad (2)$$

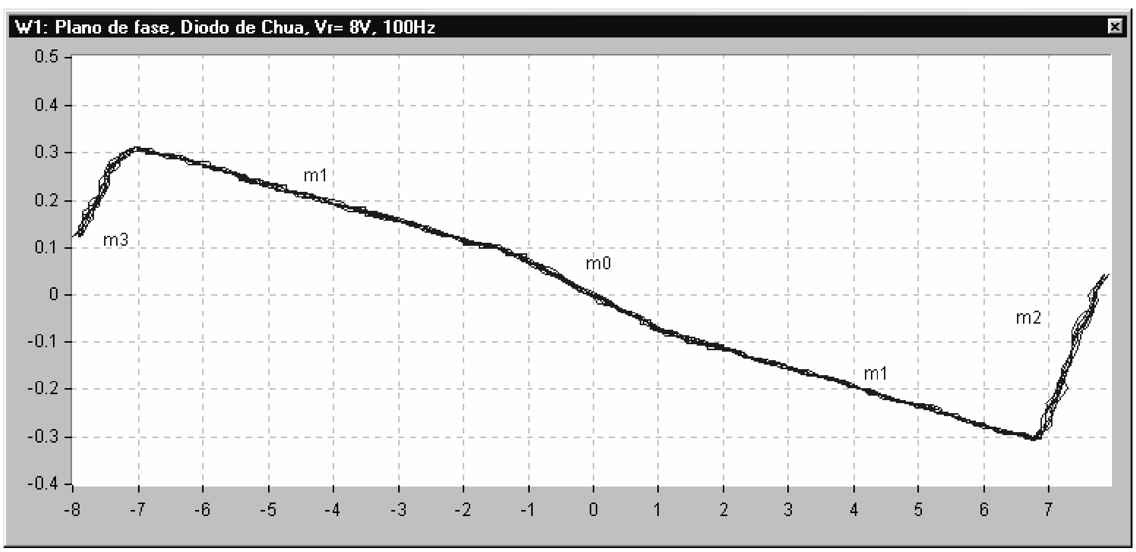


*Figure 1. Chart for the Chua diode phase, or $f(x_1)$ non-linear function, showing all slopes and breaking points, obtained according to Kennedy [4].*

where: $\alpha$= 10, $\beta$= 14.9, a= -1.27 and b= -0.68.

In figure 2, the diode and the Chua circuit are shown. In the literature there are numerous reports about these circuits [4, 6- 13].

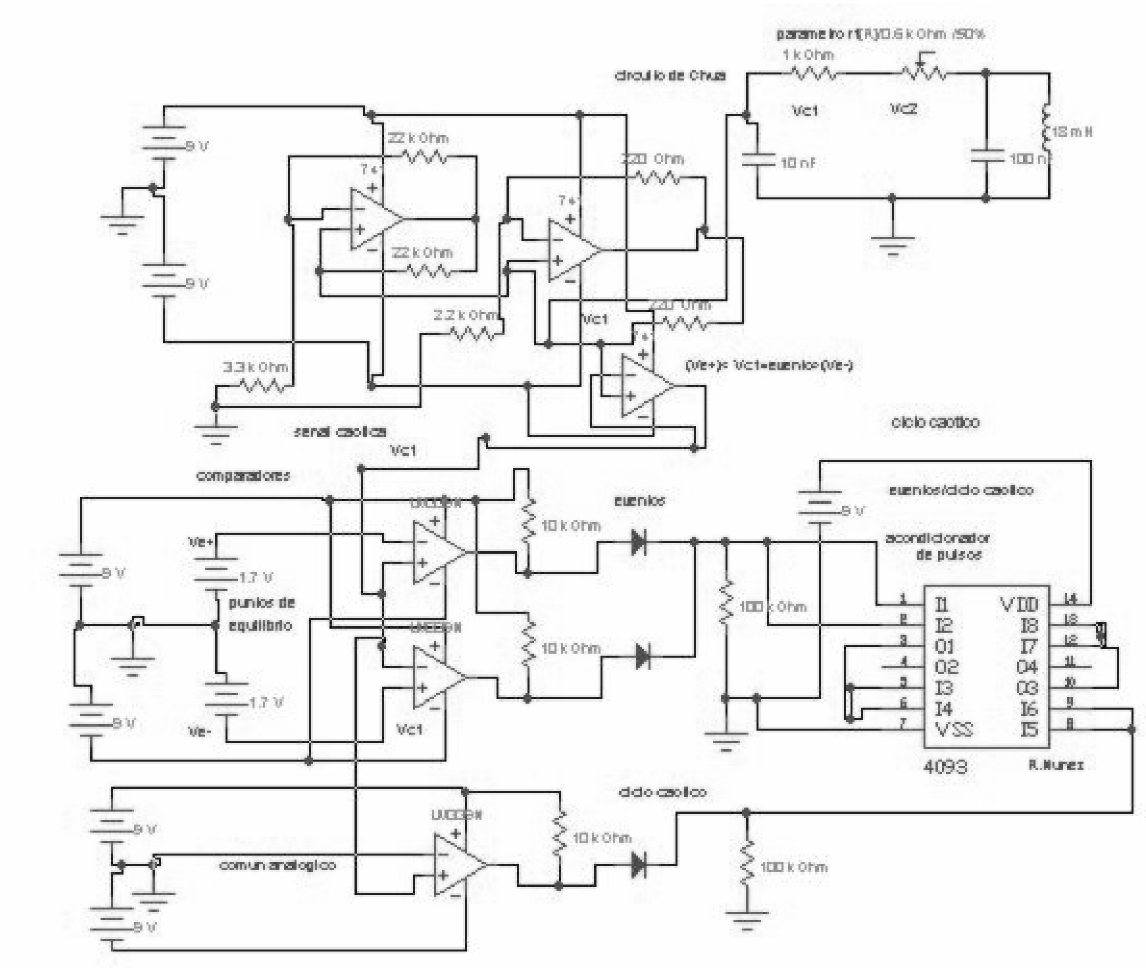


*Figure 2. Chua chaos meter circuit.*

## 3. THE CHAOS METER CIRCUIT

In figure 2, the assembled chaos meter, which consists of five circuits, is presented: Chua circuit, comparator, low-pass average filter, pulse conditioner and numerical counter/display. The first is in charge of generating the $Vc_1(x_1)$ chaos signal with a chaos dynamic imposed by an $x_1$ resistance parameter of $r_1$= 1660 ohms. The second compares the $Vc_1(x_1)$ signal magnitude with the two equilibrium levels Ve= +/- 2.0Vd.c. defined by an interval of Ve < $Vc_1(x_1)$= event > -Ve and obtained according to [4] for the case studied. *When the signal magnitude of* $Vc_1(x_1)$ *is greater than any of these Ve levels, the output of the comparator is positive and it is considered as an instantaneous event to be counted.* This action is described, graphically, in windows W4 and W5 in figure 3 (c.f., the measurements of the Dadisp Automatic Test System-DATS [14, 15] and in figure 4 (e.g., the Wb program simulation). The LM339 comparator circuit is used because it contains four of these; two are used to monitor the aforementioned equilibrium levels, and a third to generate the *base time* signal, or "*chaotic cycle"* in comparing the $Vc_1'(x_1')$ signal coming from a low-pass average filter with a common analog, -i.e., when the same crosses the zero volts level with the same slope (e.g., signifying the leaving and return to the +Ve equilibrium level) (c.f., window W3 of figure 3). This base time makes the instantaneous and particular counting of events possible, which correspond to a predetermined $r_1$ level, of the +/- Ve equilibrium voltage levels. Since this base time is always changing, the number of events will do so also, but in the very close vicinity of the middle value (c.f., figure 4, obtained in the Wb simulation program and window W4 of figure 3, already referred to, measured with the DATS). Both groups of figures represent three chaotic cycles; in that way, a representative approximation of the chaos dynamics is obtained. In the photograph in figure 5, the instantaneous measurement of a chaotic cycle of 12 events is presented in numerical displays; the number indicates the instantaneous count of the chaos dynamic of signal $Vc_1(x_1)$ for a specific chaotic cycle. Finally, the CD4093(NAND) pulse conditioning circuit, which contains four negative Y gates with Schmitt inputs, is in charge of leveling the comparator outputs to present them as a single pulse, without jittering, before the CD4510(2) numeric counter/display circuit input (not showed). The power supplies and reference voltages for the equilibrium levels are decoupled by capacitors and some techniques for depressing electric noise [16] are applied in order to make the comparator and all circuits involved more efficient.

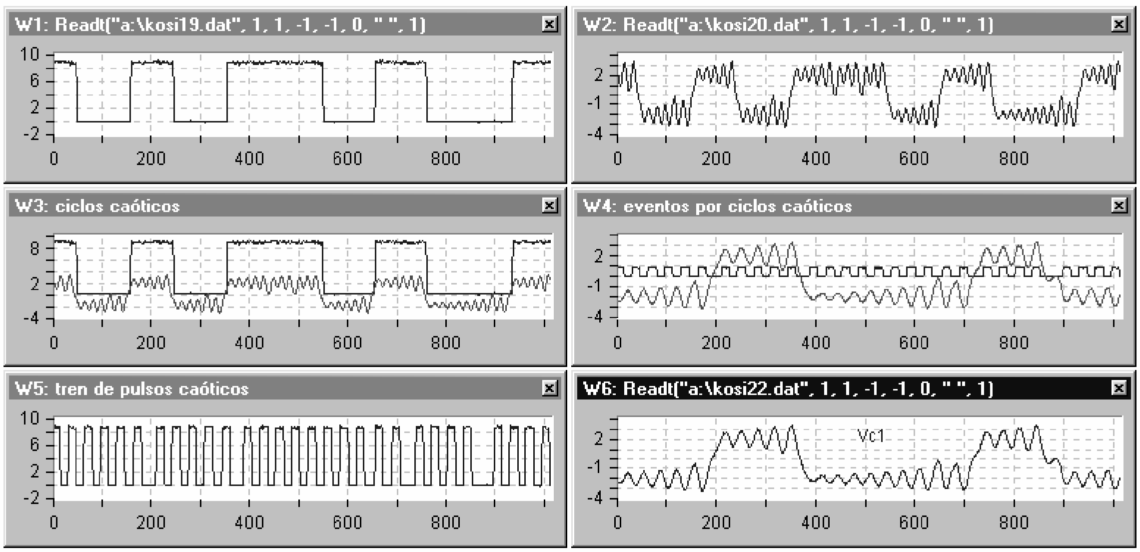


*Figure 3. Three complete chaos cycles for the $Vc_1(x_1)$ signal and its events (W3) measured with the DATS [14].*

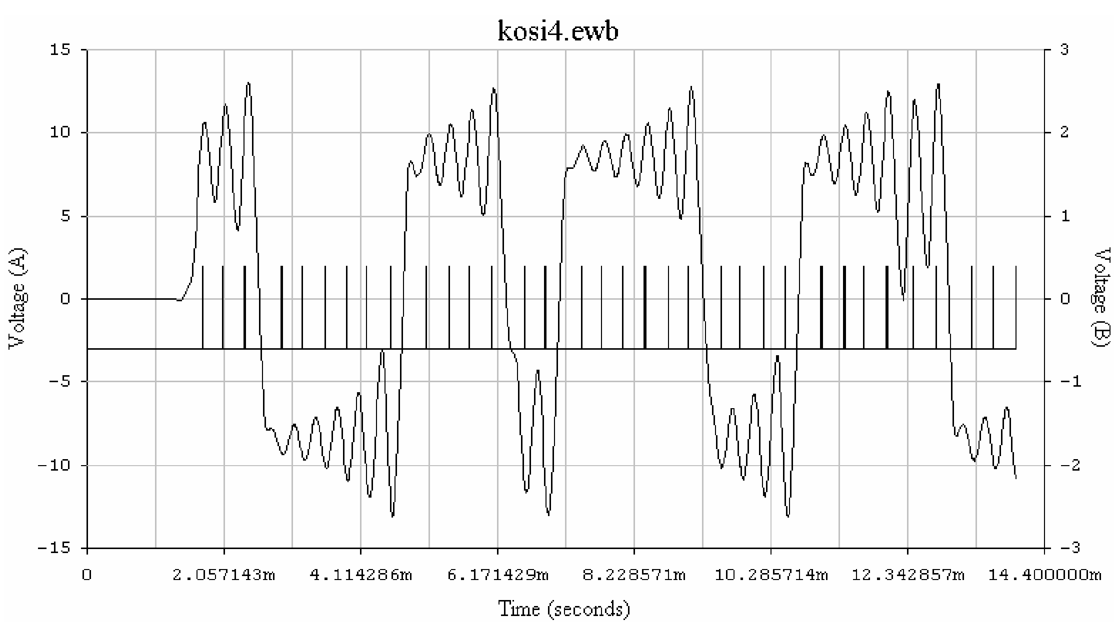


*Figure 4. Chaos signal $Vc_1(x_1)$ showing three chaotic cycles of 9, 6 and 10 events (simulation with Wb). Equilibrium levels (side B) at Ve= +/- 1.7Vd.c.*

## 4. APPLICATIONS

Applications of the constructed circuit [14, 15] (c.f., figure 5, -chaos meter circuit at real evaluation) show its capacity to quantify the number of chaotic events instantaneously, decrypt binary information, rate the synchrony of chaos circuits, and generate chaotic test sequences, among others.

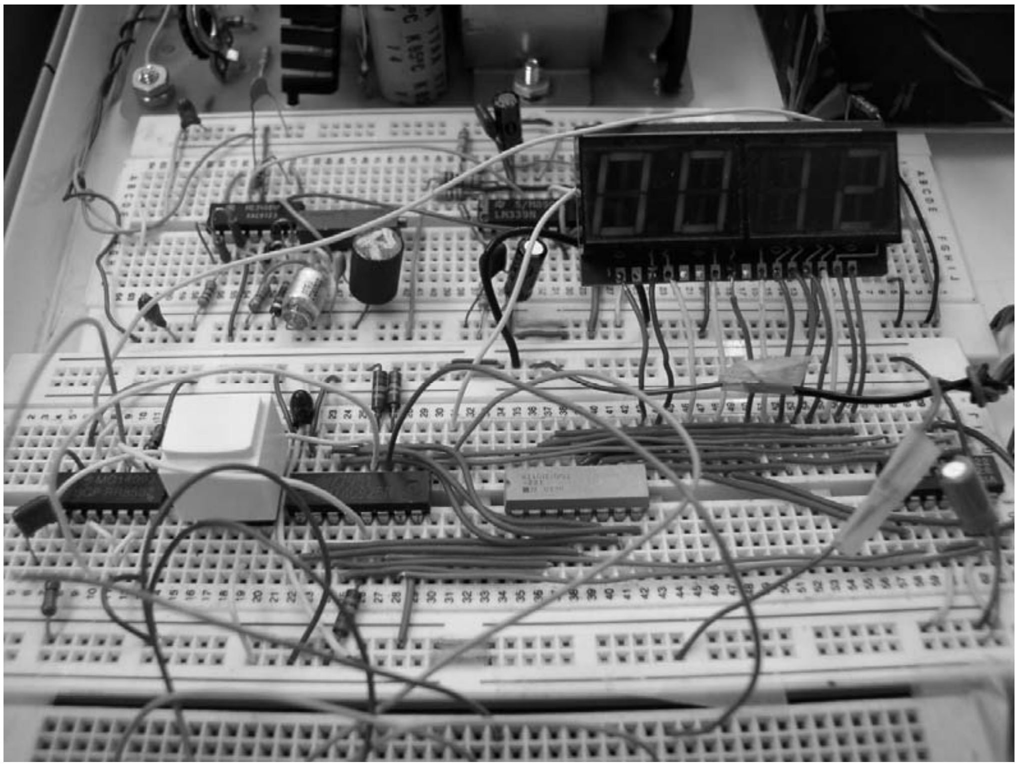

*Figure 5. Chua chaos meter displaying an instantaneous reading of 12 events or "turns" for a "chaotic cycle" of parameter $r_1$= 1660 ohms.*

### 4.1 Counting of turns or events per chaotic cycle

The idea behind this application comes from the direct relationship which exists between the waveform of signal $Vc_1(x_1)$ and its phase plane (i.e., that generated by $Vc_1(x_1)/Vc_2(x_2)$), since each time $Vc_1(x_1)$ moves from one equilibrium level to another, this amounts to -in the phase plane- moving from one scroll to another). As it is already known from Ref. 4, these position themselves in their moments at each equilibrium level as the Chua attractor graphically indicates in window W5 of figure 6. The aforementioned relationship can be clearly seen, -between the number of turns per event and the waveforms of signals $Vc_1(x_1)$ and $Vc_2(x_2)$-, shown in window W3. In this particular case, there are 7 turns on the negative event level and 4 on the positive, and 2 scroll changes as well as changes in equilibrium level are recorded. An analogy of this, but for purposes of quantification, is observed in window W4 of the same figure, where 5 turns over the negative level and 4 over the positive are registered, which amounts to a total of 9 events, or turns, per "chaotic cycle", which *number shows the instantaneous quantification of the chaos dynamic of signal $Vc_1(x_1)$ for a specific chaotic cycle.*

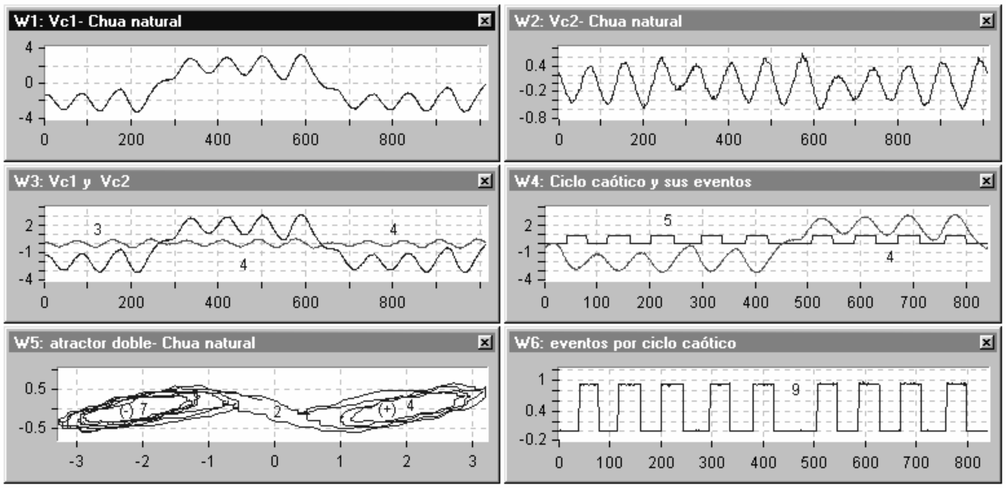


*Figure 6. Relationship of waveforms (W3) and their phase plane (W5), and their events or "turns" at the respective equilibrium level (v.g., +/- 2.0Vd.c.) (W6).*

## 4.2 Binary decryption in chaotic switching

The circuit previously referred to is applied in a decryption of hidden information process to the chaotic signal $Vc_1(x_1)$, especially when this contains binary messages sent through a line of communication and parametrically commuted [6- 10], i.e., changes in the value of the $r_1$ to $r_2$ parameters correspond to the sending of a binary state "0" or "1", respectively; in function of the waveform of the chaotic signal associated with the parameter in question, the receptor synchronizes itself and, in turn, recovers the binary status corresponding to said waveform, as long as the noise level in the channel is low. For example: parameter $r_1$, which defines a "logical zero", presents an average event count per chaotic cycle equal to 12 (c.f., with window W2 of figure 7) and diminishing the parameter by 10% of the original value, which corresponds to "logical one", -i.e., $r_2= r_1(0.90)$-. The average event count is 16 (c.f., with window W2 of figure 8), reason for which there is a difference of 4 counts, i.e., a change of 10% in the parameter represents a change of 30% in the average value corresponding to events per chaotic cycle; this indicates an acceptable sensitivity in the detection of changes. The chaotic dynamics and the events for parameter $r_2$= 1494 ohms (c.f., section 3), in figure 8.

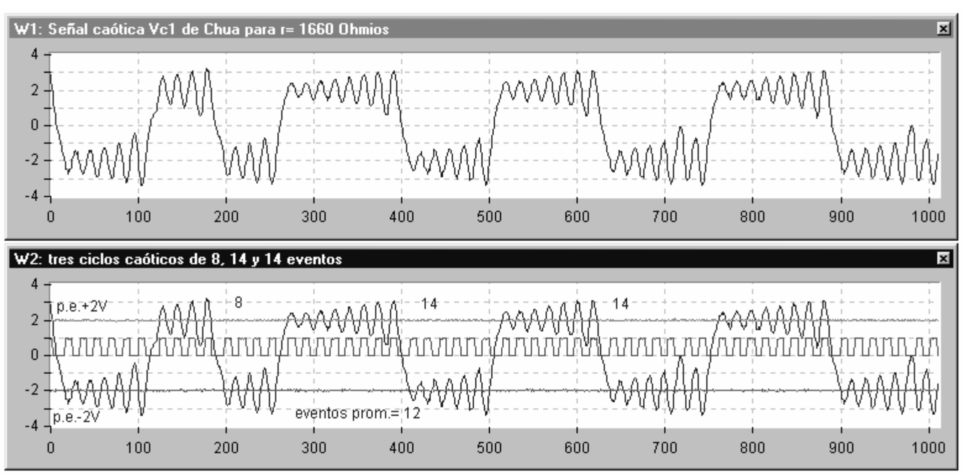


*Figure 7. Waveforms for $Vc_l$ $(x_1)$ for parameter value $r_l$= 1660 ohms. Three complete chaotic cycles are shown with 8, 14 and 14 events, with 12 of average event and equilibrium levels at +/- 2.0Vd.c.*

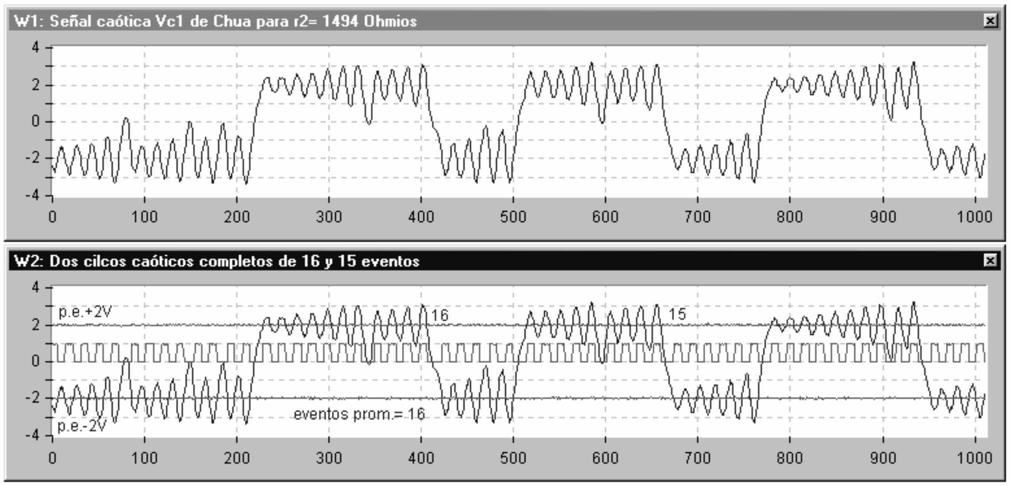


*Figure 8. $Vc_l$ ($x_1$) waveforms for parameter value of $r_2$= 1494 ohms. Two complete chaotic cycles with 16 and 15 events are shown, with equilibrium levels of +/- 2.0Vd.c.*

### 4.3 Determination of the synchrony of two identical Chua circuits

Another application of this circuit is in rating the synchrony of two identical Chua circuits coupled by the $Vc_1(x_1)$ signal [11- 13]. Here, the number of instantaneously counted events is continuously compared for the $Vc_1(x_1)$ y $Vc_1$~($x_1$~) of the transmitter and receiver, respectively. Upon the circuits being synchronized and maintained in synchrony, the counting continues in the same way (c.f., figure 7, signals $Vc_1(x_1)$ and $Vc_1$'($x_1$') and windows W1 y W2, respectively). If there is a difference, or asynchrony, -i.e., $Vc_1(x_1)$-$Vc_1$~($x_1$~)= error-, this is equivalent to a numerical difference between events in each chaotic signal, i.e., the numerical difference defines the behavior of the synchrony phase and shows whether the asynchrony is maintained during a closure. In practice, the product of the electrical noise [16] in the channel and the small variations in the parameters of the mirror circuits, the instantaneous values for the events, are very similar. Two systems of instantaneous quantifying of the $Vc_1(x_1)$ and $Vc_1$~($x_1$~) signals of the transmitter and receiver circuits were employed, respectively: a) using a single chaos meter and alternately measuring the chaos signals in question and b) using two of them and simultaneously quantifying (in phase) both signals. In both schemes the results were alike.

### 4.4 Chaotic sequences generator

Another use of the circuit is in generating chaotic test sequences, -i.e., it can be applied as a *chaotic to digital converter* (c.f., figure 4)-, etc.

## 5. CONCLUSIONS

The proposed chaos meter proved to be reliable in the *instantaneous quantification of the chaotic dynamic of the $Vc_1(x_1)$ signal of the Chua circuit*, during a "chaotic cycle", and makes the obtaining of the average chaotic dynamic during various chaotic cycles possible. It showed good average sensitivity in detecting small changes in the $Vc_1(x_1)$ waveform which are reflected on commuting the parameter, as was shown in the application of binary decryption, and which is efficient for two levels of equilibrium. In another application, it proved effective in qualifying the synchrony of two coupled Chua circuits through $Vc_1(x_1)$, comparing instantaneous quantifications of events per chaotic cycle of synchronized signals. The original interpretation of the results of the circuit constructed has to do with counting the number of turns registered by the attractor's events per chaotic cycle and, if the average of these over several chaotic cycles is calculated, we can obtain a good approximation of the number of turns. Finally, the circuit can generate chaotic sequences for testing purposes. Soon we will be preparing it to be able to count chaotic Chua circuit dynamics related to multiple scrolls and time-delay.

## 6. ACKNOWLEDGMENTS

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## Author Biography

*Ricardo F. Núñez Pérez*

Master on Science on Electronics and Instrumentation (87), CICESE, Mexico, and BS on Electronic Engineering (80), UABC, Mexico (honors at both degrees). Since 1987 to date, he is a faculty member of the Electronics & Telecommunications Department, at CICESE Research Center. He was first prize winner at the National Contest on Design of Electronics Equipment Applied to Electric Sector by IIE, CONACYT, SEP and CFE (88, 90). Research Areas: Electronics Instrument Developing for Chaotic Circuits Studies and Enchrypted Communication Applications, Industrial Machinery Vibrations and Chaotics Signals DSP Applications.