Applying Non-Steady-State Photo-Emf Technique to Detection of Higher Order Auto Correlation Functions of Ultrashort Optical Pulses

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ABSTRACT

Here we examine a new application of the adaptive detectors based on non-steady-state photo-electro-motive force effect for the detection of higher order correlation functions, aiming the estimation of the parameters of ultra short optical pulses arranged in high-repetition trains. For this purpose three beam interferometer scheme with two signal beams modulated at different frequencies is proposed. Theoretical analysis of non-steady-state photo-EMF current generated by light distribution formed by superposition of three waves is performed and the possibility to detect simultaneously second and higher order correlation function is demonstrated. Potential advantages and disadvantages of such detection scheme for measuring the higher order auto-correlations functions are discussed.

Keywords: autocorrelation function, picosecond optical pulses, photo-EMF detectors.

RESUMEN

En este trabajo se analiza una novedosa aplicación de detectores adaptivos a base del efecto de Fuerza-Foto-Electromotriz (p-EMF) en estado no estacionario para detección de las funciones de correlación de alto orden. Dicha detección tiene como objetivo la caracterización y estimación de los parámetros de trenes de pulsos ópticos ultra cortos con alta tasa de repetición. Con esta finalidad se propone un esquema experimental que utiliza un interferómetro de tres haces con dos de ellos modulados temporalmente en fase. Posteriormente se presenta el análisis teórico de la corriente p-EMF generada por la distribución-intensidad formada en la superposición de tres haces que incidan en detectores bajo las condiciones experimentales antes mencionadas. Los resultados de este análisis demuestra la posibilidad de detectar simultáneamente la función de correlación de segundo y de cuarto orden. Finalmente se discuten las ventajas y desventajas potenciales de la detección de funciones de auto correlación utilizando los detectores de p-EMF.

1. Introduction

Non-steady-state photo-EMF effect [1] manifests itself as alternating current (ac) flowing through the short circuited photoconductive sample illuminated by oscillating periodical light distribution created by e.g. superposition of two coherent light beams (signal S and reference R). This ac current appears due to the interaction between the distribution of the mobile carriers $n(x)$ excited by light and the pattern of space charge electric field $E_{sc}(x)$ created by charges trapped on deep trapping centers. Using non steady photo-EMF phase sensitive photodetectors as an integrated optical interferometer for the detection of optical signal modulated in phase is quite a breakthrough idea being under development for more than

decade. The inherent adaptive (spatial and temporal) properties of such detectors make them promising e.g. for nondestructive testing of vibrations from rough surfaces in industrial environment. Due to its robustness, simplicity, and high sensitivity the photo-EMF based configuration was proposed for numerous practical applications such as laser vibrometry [2], laser induced ultrasonic detection [3], profilometry of rough surfaces [4], etc. [5]-[7].

One potential application of photo-EMF detector can be associated with the measurements of interference pattern visibility m [5]-[7], which makes possible evaluation of mutual coherence of two waves $\Gamma_{R,S}$. The basic idea of this application lays in the fact that for all other experimental parameters (wavelength, amplitude and frequency

of periodical modulation, period of interference patterns, light intensity, etc.) being fixed the amplitude of the photo-EMF signal is proportional to the square of interference pattern visibility $\propto m^2$. Hence, it must be proportional to the second power of a degree of mutual temporal coherence between two interfering beams, because in general [8],

$$m\left(g_{R,S}\right) = \frac{\sqrt{I_R I_S}}{I_0} \Gamma_{R,S}\left(\tau\right) \qquad (1)$$

where $I_{R,S}$ are the intensities of reference and signal beams respectively, $I_0 = I_R + I_S$ is the average light intensity and τ is the delay time. Ding et al. [5], demonstrated experimentally that measuring photo-EMF output signal amplitude as a function of a delay time τ it is possible to evaluate the second power of autocorrelation functions of the electric field envelope of the laser pulses.

Here we explore the possibility to use photo-EMF detector for measuring of higher order correlation functions aiming the evaluation of the parameters of ultra short optical pulses. The interest on higher order (triple, forth, etc.) correlation functions is motivated by the fact that it can provide more information about the pulse parameters, than it does the standard second order autocorrelation function, e.g. it can be helpful for retrieving the information about the phase of the pulse [9].

The subsequent sections are organized in the following way. As starting point, the brief description of working principle of photo-EMF detector for the measurement of second order correlation function using two beams interferometer is presented (Ding et al. [5]). Next, theoretical analysis of non-steady-state photo-EMF current generated by the detector placed in three beams interferometer is performed and the possibility to detect simultaneously second and higher order correlation functions is demonstrated. Finally, the advantages and disadvantages of this new technique for measuring pulse auto-correlation functions are discussed.
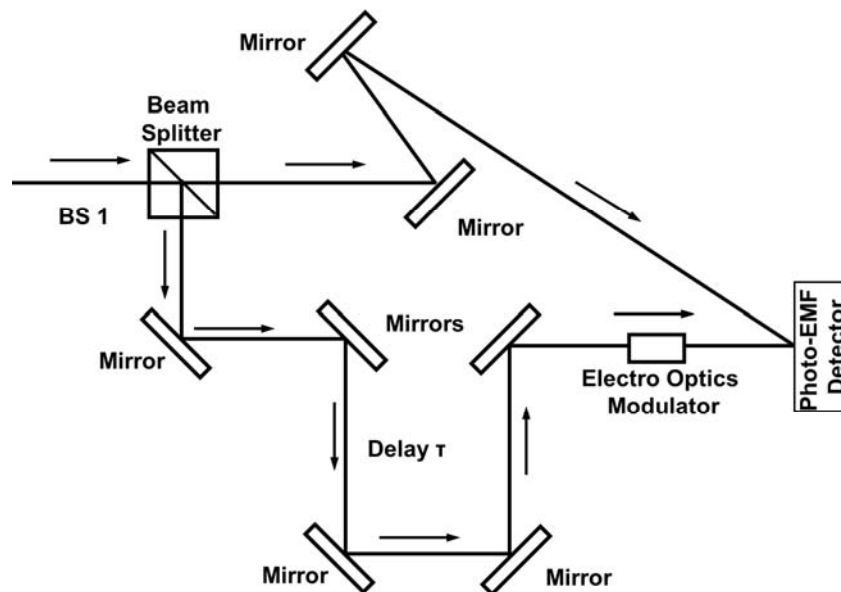

Figure 1. Typical experimental arrangement of the two Gaussian pulse correlator based on non-steady-state photo-EMF effect.

2. Main aspects of the photo-Emf technique for detecting the second order autocorrelation functions

Detailed analysis of two Gaussian pulse correlator using a non-steady-state photo-EMF detector can be found in Ref. [5]. Here we present only a brief description of the working principle of photo-EMF based correlator. Basic experimental set up (Fig. 1) for such correlator consists of two beam interferometer with temporal delay among them and additional periodical phase modulation in one of the arms.

The resulting interference pattern is brought on the surface of the photo-EMF detector, and the resulting ac current is measured by lock-in amplifier.

Let us consider two laser beams (reference and signal), consisting on a sequence of ultra short pulses entering a detector with an incidence angle $\theta = \theta_S = \theta_R$ and a time delay τ on the reference beam. The total electric field on a detector can be written as

$$E(t,\tau) = E_S(t) + E_R(t+\tau) = E_R f_R(t+\tau) \exp\left[i\left(k_R z_0 + \psi(t)\right)\right] +$$
$$E_S f_S(t) \exp\left[i\left(k_S z_0 + \psi(t) + \psi(t+\tau)\right)\right], \tag{2}$$

with field envelopes $f_S(t)$ and $f_R(t+\tau)$. For a Gaussian pulse the expression for electric field temporal envelopes can be written as:

$$f_R(t) = \exp\left(-t/T_R^2\right), \qquad f_S(t+\tau) = \exp\left(-(t+\tau)/T_S^2\right), \tag{3}$$

where T_S or T_R is the full width at half-maximum (FWHM) of the signal and reference pulse respectively. The corresponding light intensity distribution is:

$$I(t,\tau) = I_R A + I_S B + \sqrt{I_R I_S}\, g_{SR}(\tau) \exp(-i\omega_L \tau) \exp\left\{i\left[Kz - \psi(t+\tau)\right]\right\} + c.c. \tag{4}$$

where $K = 2\pi / \Lambda = 2k\sin(\theta)$ is the amplitude of the grating vector $\vec{K} = \vec{k}_R - \vec{k}_S$. and A, B and $g_{S,R}(\tau)$ .are defined as:

$$a)\ A = f_R^2(t) \qquad b)\ B = f_S^2(t+\tau). \quad \text{and} \quad c)\ g_{R,S}(\tau) = f_R(t) f_S^*(t+\tau) \tag{5}$$

Since electronic detector is too slow comparing with electric field oscillating at optical frequency, the intensity seen by the detector is the time averaged:

$$I(\tau) = \int_\infty^\infty I(t,\tau)\, dt = I_0 \left\{1 + \frac{1}{2} m_0 \Gamma_{S,R} \exp(i\omega_L \tau) \exp(iKz_0) + c.c.\right\}, \tag{6}$$

where I_0 is the total intensity, $I_0 = I_R \Gamma_R + I_S \Gamma_S$, with average reference I_R and signal I_S intensity, and Γ_R and Γ_S are constants which for a Gaussian pulses can be easily calculated:

$$a)\ \Gamma_R = \int_{-\infty}^\infty f_R(t)^2\, dt = \sqrt{\frac{\pi}{2}} T_R, \quad b)\ \Gamma_S = \int_{-\infty}^\infty f_S(t)^2\, dt = \sqrt{\frac{\pi}{2}} T_S. \tag{7}$$

One can see that the time varying modulation index of the interference pattern $m(\tau) = m_0 \Gamma_{S,R}$ with the nominal modulation $m_0 = 2\sqrt{I_S I_R}/I_0$, depends directly to the electric-field correlation function $\Gamma_{R,S}(\tau)$ of the two beams which is given by

$$\Gamma_{R,S}(\tau) = \int_{-\infty}^{\infty} f_R(t) f_S^*(t+\tau) dt. \qquad (8)$$

For Gaussian beam:

$$\Gamma_{R,S}(\tau) = \frac{(T_S^2 + T_R^2)\pi}{2} \exp\left(-\frac{\tau^2}{T_S^2 + T_R^2}\right). \qquad (9)$$

If the phase of one of the beams is modulated sinusoidally with amplitude Δ and frequency Ψ, the expression for phase temporal dependence becomes

$$\psi(t) = \Delta \sin(\Psi t). \qquad (10)$$

Such oscillating interference pattern gives rise to photo-EMF current flowing through the sample. In order to calculate its value the standard set of equation (given e.g. in Ref. [1]) should be solved in approximation of small contrast ($m \ll 1$) of the interference pattern and small amplitude of vibrations $\Delta \ll 1$. As a result the expression for amplitude of first harmonic of photo-EMF current density can be obtained:

$$J^\Psi(t) = \frac{m^2(g_{R,S})\Delta}{2}\sigma_0 E_D \frac{i\Psi\tau_{di}}{1+i\Psi\tau_{di}(1+K^2 L_D^2)} \qquad (11)$$

where the contrast of the interference pattern is given by Eq.1, E_D is the diffusion field $L_D = \sqrt{D\tau_c}$ is the diffusion length with diffusion coefficient D and carrier lifetime τ_c, σ_0 is the average photoconductivity of the sample, and $\tau_{di} = \varepsilon\varepsilon_0/\sigma_0$ is the dielectric relaxation time with effective dielectric constant $\varepsilon\varepsilon_0$

One can see that the first harmonic of photo-EMF signal is proportional to the square of the second order correlation function $\Gamma_{S,R}(\tau)$
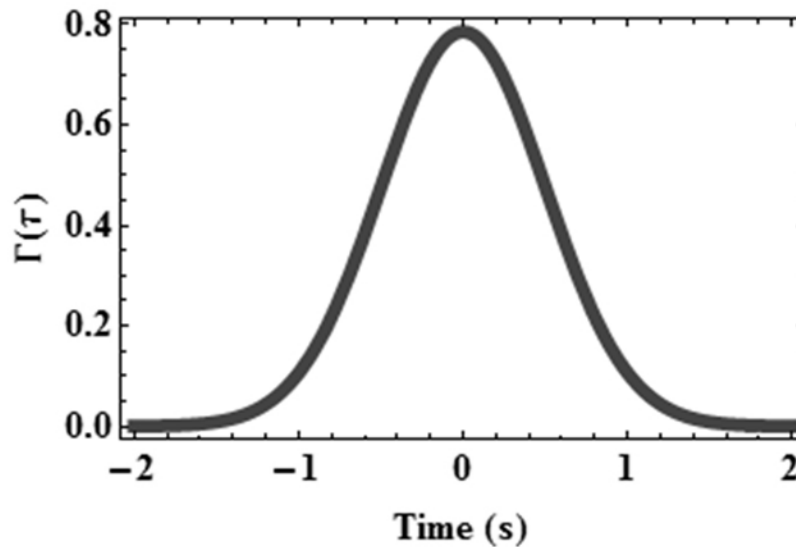
$$J^\Psi(t) \propto \Gamma_{R,S}^2(\tau) \qquad (12)$$

Figure 2. Example of the pulse correlation function obtained with the
Photo-EMF detector with **T=1**.

By measuring the photo-EMF current as a function of the time delay of optical pulses, one can deduce information about the signal pulses, such as pulse width, given a known reference pulse. This method gives the same information as the interferometric electric-field correlation does but without the need for data processing. In the case of transform limited pulses the photo-EMF current trace as a function of the delay is identical to the conventional background-free intensity correlation, which gives the pulse width directly as it is showing in the fig. 2

3. Theoretical analisis of photo-EMF technique using three beam interferometer for detection of higher order correlation function

Here we develop the theory of the photo-EMF [1] applied to detection of higher order correlation function of Gaussian pulses. The proposed configuration is based in the three beams interferometer (Fig. 3).

In general, the three beam Gaussian pulse correlator is based in a modified Michelson interferometer. The triplet arms of this three beam interferometer provide the scanning of the pulse trains. The beam from the source is divided in two beams by the beam splitter. One of them it is arriving to the Moving Mirror 1 where it is delayed in time $I(t + \tau_1)$ and reflected. The other beam again is divided in two parts. The second part of the beam arrive to the moving mirror 2 where it is delaied $I(t + \tau_2)$ and reflected. The last beam with the intensity $I(t)$ comes in to the fixed mirror. Periodical phase modulation can be introduced in first two beams e.g. by attaching the mirror in the delay line to the piezoelectric transducer. Finally, three pulse trains are mixed on photo-EMF detector, whose output current is detected by lock-in amplifier.
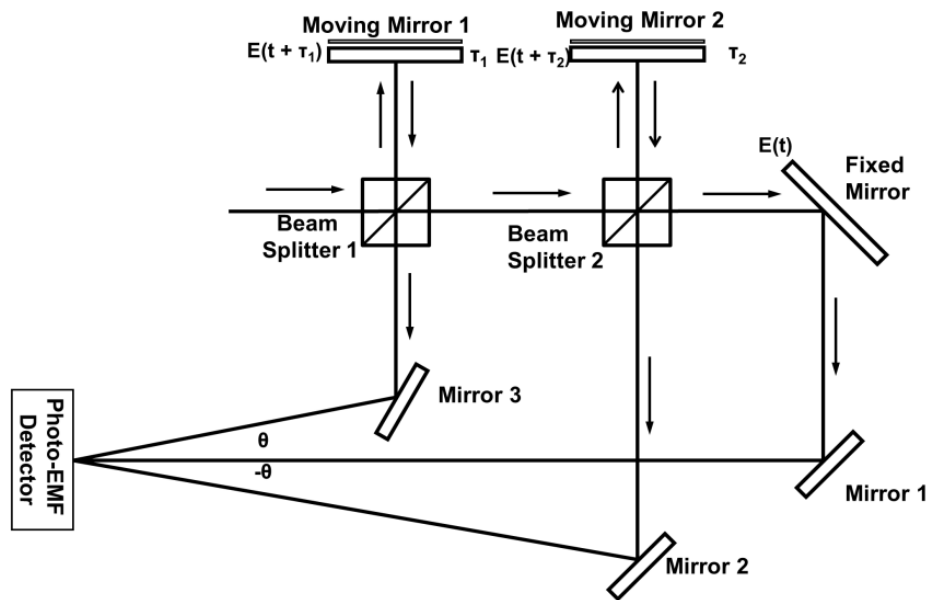

Figure 3. Experimental arrangement of the three beam correlator based
on non-staeady-state photo-EMF effect.

As in the previous section we consider Gaussian pulses:

$$a)\, E_1(t) = E_1 f_1(t) \exp\left[i\left(k_1 z_0 + \psi(t)\right)\right],$$

$$b)\, E_2(t + \tau_1) = E_2 f_2(t + \tau_1) \exp\left[i\left(k_2 z_0 + \psi(t) + \psi(t + \tau_1)\right)\right],.$$

$$c)\, E_3(t + \tau_2) = E_3 f_3(t + \tau_2) \exp\left[i\left(k_3 z_0 + \psi(t) + \psi(t + \tau_2)\right)\right],$$

(13)

where $E_{1,2,3}$ and $\vec{k}_{1,2,3}$. are the electric fields and the wave vectors respectively, $\psi(t)$, $\psi(t + \tau_1)$. and $\psi(t + \tau_2)$ are the respectively phase function. The functions $f_{1,2,3}$ are defined as

$$a)\, f_1(t) = \exp\left(-\frac{t^2}{T^2}\right), \qquad b)\, f_2(t + \tau_1) = \exp\left[-\frac{(t + \tau_1)^2}{T^2}\right],$$

$$c)\, f_3(t + \tau_2) = \exp\left[-\frac{(t + \tau_2)^2}{T^2}\right],$$

(14)

where $\tau_{1,2}$ are the respectively pulses time delay [6]. For the simplicity it is assumed that all pulses possess the same FWHM. Then the interference pattern appeared as a result of superposition of three beams is given by the following expression:

$$I_T(t) = A I_1 + B I_2 + C I_3 + \sqrt{I_1 I_2}\, g_{1,2} \exp\left\{i\left[K_0 z_0 - \psi(t + \tau_1)\right]\right\} +$$

$$+ \sqrt{I_1 I_3}\, g_{1,3} \exp\left\{-i\left[K_0 z_0 + \psi(t + \tau_2)\right]\right\} +$$

$$+ \sqrt{I_2 I_3}\, g_{2,3} \exp\left(i\left\{2 K_0 z_0 - \left[\psi(t + \tau_1) - \psi(t + \tau_2)\right]\right\}\right) + C.C.$$

(15)

where $\vec{K}_1 = \vec{k}_2 - \vec{k}_1.$, $\vec{K}_2 = \vec{k}_3 - \vec{k}_1$ and $\vec{K}_3 = \vec{k}_2 - \vec{k}_3$. Experimentally it is possible arrange the incident beams in such a way that $\vec{K}_1 = \vec{K}_0$, $\vec{K}_2 = -\vec{K}_0$ and $\vec{K}_3 = 2\vec{K}_0$. As before, the functions $g_{i,j}$ and A, B, C are defined as

$$g_{1,2} = f_1(t) f_2^*(t + \tau_1), \quad g_{1,3} = f_1(t) f_3^*(t + \tau_2), \quad g_{2,3} = f_2(t + \tau_1) f_3^*(t + \tau_2).$$

(16)

$$A = \left|f_1(t)\right|^2, \qquad B = \left|f_2(t + \tau_1)\right|^2, \qquad C = \left|f_3(t + \tau_2)\right|^2.$$

(17)

If each of the signal beams is modulated sinusoidally at different frequencies Ψ_1 and Ψ_2, their phase temporal dependencies can be expressed as

$$\psi(t) = \Delta \sin(\Psi_1 t) \text{ and } \psi(t) = \Delta \sin(\Psi_2 t).$$

(18)

Following the same procedure as that described in the last session one can obtain the equation of the photo-EMF current density produced by the interferences of the three beams has four components corresponding to the frequencies Ψ_1, Ψ_2, $\Psi_1 \pm \Psi_2$

$$\text{a) } J_{p-emf}^{\Psi_1}(t) = \frac{m^2(\Gamma_{1,2})\Delta}{2}\sigma_0 E_D \frac{i\Psi_1 \tau_{di}}{1+i\Psi_1 \tau_{di}(1+K_2 L_D^2)}$$

$$\text{b) } J_{p-emf}^{\Psi_2}(t) = \frac{m^2(\Gamma_{1,3})\Delta}{2}\sigma_0 E_D \frac{i\Psi_2 \tau_{di}}{1+i\Psi_2 \tau_{di}(1+K_2 L_D^2)}$$

$$\text{c) } J_{p-emf}^{\Psi_1-\Psi_2}(t) = \frac{M(\Gamma_{1,2},\Gamma_{2,3})\Delta}{2} \frac{\sigma_0 E_D i(\Psi_1 - \Psi_2)\tau_{di}}{1+i(\Psi_1 - \Psi_2)\tau_{di}(1+K_2 L_D^2)} \qquad (19)$$

$$\text{d) } J_{p-emf}^{\Psi_1+\Psi_2}(t) = \frac{M(\Gamma_{1,2},\Gamma_{2,3})\Delta}{2} \frac{\sigma_0 E_D i(\Psi_1 + \Psi_2)\tau_{di}}{1+i(\Psi_1 + \Psi_2)\tau_{di}(1+K_2 L_D^2)},$$

where

$$m(\Gamma_{1,2}) = \sqrt{I_1 I_2}/I_0 \, \Gamma_{1,2} = m_{1,2}\Gamma_{1,2}, \qquad m(\Gamma_{1,3}) = \Gamma_{1,3}\sqrt{I_1 I_3}/I_0 = m_{1,3}\Gamma_{1,3}, \qquad (20)$$

$$M(\Gamma_{1,2,1,3}) = \sqrt{I_1 I_2 I_1 I_3}/I_0^2 \, \Gamma_{1,2,1,3} = m_{1,2}m_{1,3}\Gamma_{1,2,1,3}.$$

And

$$\text{a) } \Gamma_{1,2} = \int_{-\infty}^{\infty} f_1(t)f_2^*(t+\tau_1)dt = \frac{T^2\pi}{2}\exp\left(-\frac{\tau_1^2}{T^2}\right),$$

$$\text{b) } \Gamma_{1,3} = \int_{-\infty}^{\infty} f_1(t)f_3^*(t+\tau_2)dt = \frac{T^2\pi}{2}\exp\left(-\frac{\tau_2^2}{T^2}\right), \qquad (21)$$

$$\Gamma_{1,2,1,3} = \int_{-\infty}^{\infty} |f_1(t)|^2 f_2(t+\tau_1)f_3^*(t+\tau_2)dt = \frac{T}{2}\sqrt{\pi}\exp\left[-\frac{3}{4T^2}\left(\tau_1^2 + \tau_2^2 - \frac{2}{3}\tau_1\tau_2\right)\right], \qquad (22)$$

First two terms (Eq. 19 a and b) carry the same information as in the case of two-pulse correlator discussed in the previous section and gives the possibility to measure the square of the second order electric field correlation function.

As to the last two terms, they are proportional to the higher order correlation function, namely the 4-th order correlation function [6]. Then one can calculated the corresponding bispectrum which is given by

$$\Gamma_{DG}(f_1, f_2) = m_{0_{12}} m_{0_{13}} T\sqrt{\frac{\pi}{2}}\exp\left[-\frac{3\pi^2 T^2}{2}\left(f_1^2 + f_2^2 - \frac{2}{3}f_1 f_2\right)\right]. \qquad (23)$$

Thus, the proposed three beam interferometer allows to measure both second and higher order correlation function in a single experimental arrangement using non-steady-state photo-EMF phase sensitive detector. The main advantage of this technique (compared with the conventional measurements of pattern contrast) is the simplicity of the configuration and the potentially high speed of operation. Indeed, the output signal from the lock-in amplifier gives the value of the mutual coherence directly without any signal processing.

Characteristic time which is needed to measure the value of autocorrelation function for a particular optical path is basically limited by the characteristic time of space charge grating formation which can be as small as 10^{-7} s in GaAs detector in the visible and infrared region of spectra. Other important advantage of this technique consists on the possibility to work with complicated wavefronts, even with speckle-like pattern instead of perfect Gaussian interfering wavefronts. Finally, unlike standard interferometer photo-EMF base correlator can operate in a presence of environmental vibration and noises.

Among the disadvantage, one can mention that the detected correlation function is of the even order, which make impossible the extraction of information about the phase of the pulses. Another limitation of photo-EMF based detector is that the distance between electrodes should be small enough; in particular the interelectrode spacing should fit the maximal number of fringes which can be produced for given experimental condition, this last one, in general, is defined by the ratio $\lambda / \Delta\lambda$ of the source

4. Conclusions

We propose the methodology to measure higher order correlation functions of short optical pulses using non-steady-state photo-EMF detector. For this purpose three beam interferometer scheme with two signal beams modulated at different frequency is proposed. Theoretical analysis of non-steady-state photo-EMF current generated by light distribution formed by superposition of three waves is performed and the possibility to detect simultaneously second and higher order correlation function is demonstrated.
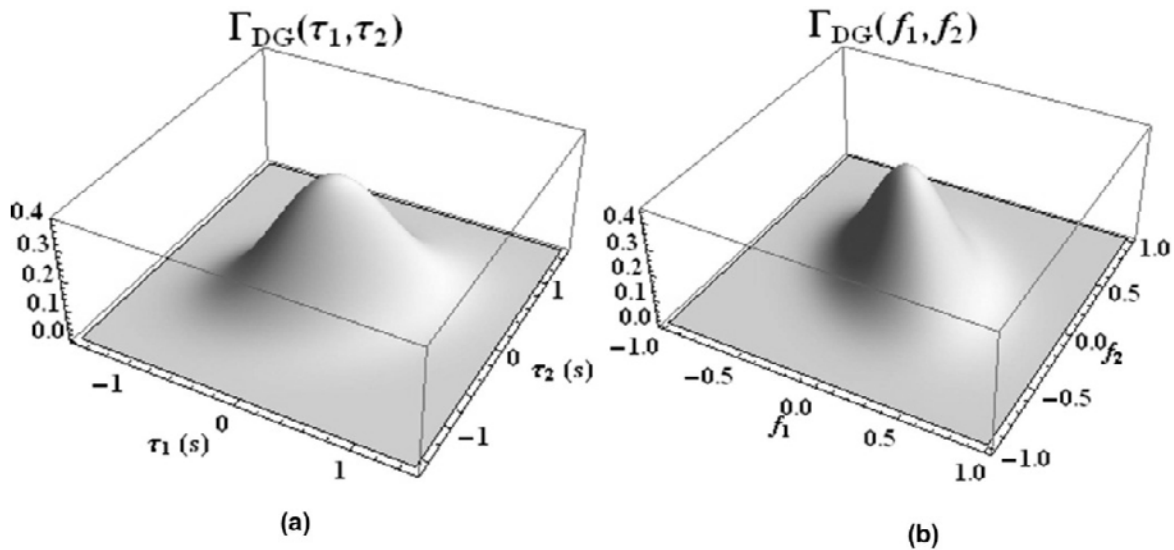

Figure 4. (a) Correlation function calculated from Eq. 22 and (b) its bispectrum (Eq. 23); T=1.

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