

General Form of Linear Programming Problems with Fuzzy Parameters

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ABSTRACT

In this paper, it is pointed out that the existing general form of such fully fuzzy linear programming problems in which all the parameters are represented by such flat fuzzy numbers for which is valid only if there is not a negative sign. However, if there is a negative sign, then the existing general form of fully fuzzy linear programming problems is not valid. Thus, a new general form is proposed.

Keywords: fuzzy parameters, LR flat fuzzy numbers, fuzzy linear programming.

1. Introduction

Linear programming is one of the most frequently applied operation research techniques. Although it has been investigated and expanded for more than six decades by many researchers and from various points of view, it is still useful to develop new approaches in order to fit better real-world problems within the framework of linear programming.

In conventional approach, parameters of linear programming models must be well defined and precise. However, in a real-world environment, this is not a realistic assumption. Usually, the value of many parameters of a linear programming model is estimated by experts. Clearly, it cannot be assumed that the knowledge of experts is precise enough. Bellman and Zadeh [2] proposed the concept of decision making in fuzzy environments. After that, a number of researchers have exhibited their interest to solve the fuzzy linear programming problems [1, 5-30].

In this paper, the shortcomings of existing general form of fully fuzzy linear programming problems are pointed out and a new general form of fully fuzzy linear programming problems is proposed.

This paper is organized as follows: In Section 2, some basic definitions and arithmetic operations are presented. In Section 3, the shortcomings of existing general form of fully fuzzy linear programming problems are pointed out. In Section 4, a new general form of fully fuzzy linear programming

problems is proposed and the advantages of the proposed form over the existing form are discussed. Conclusions are discussed in Section 5.

2. Preliminaries

In this section, some basic definitions and arithmetic operations are presented [20].

2.1 Basic definitions

Definition 2.1: [4] A function, usually denoted by $L : [0, \infty) \rightarrow [0, 1]$ or $R : [0, \infty) \rightarrow [0, 1]$ is said to be the reference function of fuzzy number if and only if (i) $L(0) = 1$ (ii) L is nonincreasing in $[0, \infty)$.

Definition 2.2: [4] A fuzzy number \tilde{A} defined on the set of real numbers R , denoted as $\tilde{A} = (m, n, \alpha, \beta)_{LR}$ is said to be an LR flat fuzzy number if

$$\mu_{\tilde{A}}(x) = \begin{cases} L\left(\frac{m-x}{\alpha}\right), & x \leq m \\ R\left(\frac{x-n}{\beta}\right), & x \geq n \\ 1, & \text{otherwise} \end{cases}$$

Where, $\alpha > 0, \beta > 0$.

Definition 2.3: [4] Let $\tilde{A} = (m, n, \alpha, \beta)_{LR}$ be an *LR* flat fuzzy number and λ be a real number in the interval $[0, 1]$ then the crisp set, $A_\lambda = \{x \in X : \mu_{\tilde{A}}(x) \geq \lambda\} = [m - \alpha L^{-1}(\lambda), n + \beta R^{-1}(\lambda)]$, is said to be λ -cut of \tilde{A} .

Definition 2.4: [3] An *LR* flat fuzzy number $\tilde{A} = (m, n, \alpha, \beta)_{LR}$ is said to be non-negative *LR* flat fuzzy number if $m - \alpha \geq 0$ and is said to be nonpositive *LR* flat fuzzy number if $n + \beta \leq 0$.

2.2 Arithmetic operations

In this section, the arithmetic operations between *LR* flat fuzzy numbers are presented [4].

Let $\tilde{A}_1 = (m_1, n_1, \alpha_1, \beta_1)_{LR}$, $\tilde{A}_2 = (m_2, n_2, \alpha_2, \beta_2)_{LR}$ be two *LR* flat fuzzy numbers and $\tilde{A}_3 = (m_3, n_3, \alpha_3, \beta_3)_{RL}$ be a *RL* flat fuzzy number.

Then

$$\begin{aligned}\tilde{A}_1 \oplus \tilde{A}_2 &= (m_1 + m_2, n_1 + n_2, \alpha_1 + \alpha_2, \beta_1 + \beta_2)_{LR} \\ \tilde{A}_1 \ominus \tilde{A}_3 &= (m_1 - n_3, n_1 - m_3, \alpha_1 + \beta_3, \beta_1 + \alpha_3)_{LR}\end{aligned}$$

If \tilde{A}_1 and \tilde{A}_2 both are non-negative, then

$$\tilde{A}_1 \otimes \tilde{A}_2 = (m_1 m_2, n_1 n_2, m_1 \alpha_2 + \alpha_1 m_2 - \alpha_1 \alpha_2, n_1 \beta_2 + \beta_1 n_2 + \beta_1 \beta_2)_{LR}$$

If \tilde{A}_1 is nonpositive and \tilde{A}_2 is non-negative, then

$$\tilde{A}_1 \otimes \tilde{A}_2 = (m_1 n_2, n_1 m_2, \alpha_1 n_2 - m_1 \beta_2 + \alpha_1 \beta_2, \beta_1 m_2 - n_1 \alpha_2 - \beta_1 \alpha_2)_{LR}$$

If \tilde{A}_1 is non-negative and \tilde{A}_2 is nonpositive, then

$$\tilde{A}_1 \otimes \tilde{A}_2 = (n_1 m_2, m_1 n_2, n_1 \alpha_2 - \beta_1 m_2 + \beta_1 \alpha_2, m_1 \beta_2 - \alpha_1 n_2 - \alpha_1 \beta_2)_{LR}$$

If \tilde{A}_1 and \tilde{A}_2 both are nonpositive, then

$$\tilde{A}_1 \otimes \tilde{A}_2 = (n_1 n_2, m_1 m_2, -n_1 \beta_2 - \beta_1 n_2 - \beta_1 \beta_2, -m_1 \alpha_2 - \alpha_1 m_2 + \alpha_1 \alpha_2)_{LR}$$

and scalar multiplication is defined as

$$\lambda \tilde{A}_1 = \begin{cases} (\lambda m_1, \lambda n_1, \lambda \alpha_1, \lambda \beta_1)_{LR} & \lambda \geq 0 \\ (\lambda n_1, \lambda m_1, -\lambda \beta_1, -\lambda \alpha_1)_{LR} & \lambda < 0 \end{cases}$$

3. Shortcomings of existing general form of fully fuzzy linear programming problems

In the existing methods [1, 12-17] it is assumed that the general form of fully fuzzy linear programming problems (P_2) is obtained by replacing the crisp parameters c_j , a_{ij} , b_i and x_j of crisp linear programming problem (P_1) by fuzzy parameters \tilde{c}_j , \tilde{a}_{ij} , \tilde{b}_i and \tilde{x}_j respectively.

$$\begin{aligned}\text{Maximize/Minimize } & \left(\sum_{j \in N_1} c_j x_j - \sum_{j \in N_2} c_j x_j \right) \\ \text{Subject to} & \quad (P_1)\end{aligned}$$

$$\sum_{j \in N_3} a_{ij} x_j - \sum_{j \in N_4} a_{ij} x_j \leq, =, \geq b_i, \quad i = 1, 2, \dots, m$$

Where, x_j , a_{ij} , b_i , c_j are any real numbers and $N_1 \cup N_2 = \{1, 2, \dots, n\}$, $N_3 \cup N_4 = \{1, 2, \dots, n\}$, $N_1 \cap N_2 = \phi$, $N_3 \cap N_4 = \phi$.

$$\begin{aligned}\text{Maximize/Minimize } & \left(\sum_{j \in N_1} \tilde{c}_j \otimes \tilde{x}_j \ominus \sum_{j \in N_2} \tilde{c}_j \otimes \tilde{x}_j \right) \\ \text{Subject to} & \quad (P_2)\end{aligned}$$

$$\sum_{j \in N_3} \tilde{a}_{ij} \otimes \tilde{x}_j \ominus \sum_{j \in N_4} \tilde{a}_{ij} \otimes \tilde{x}_j \leq_{\Re}, =_{\Re}, \geq_{\Re} \tilde{b}_i, \quad i = 1, 2, \dots, m$$

Where, \tilde{x}_j , \tilde{a}_{ij} , \tilde{b}_i , \tilde{c}_j are unrestricted fuzzy numbers and $N_1 \cup N_2 = \{1, 2, \dots, n\}$, $N_3 \cup N_4 = \{1, 2, \dots, n\}$, $N_1 \cap N_2 = \phi$, $N_3 \cap N_4 = \phi$.

However, if all the parameters of (P_2) are represented by LR flat fuzzy numbers then it is not genuine to use the fully fuzzy linear programming problem (P_2) to find the fuzzy optimal solution of real life problems due to the following reasons:

In previous studies, it was pointed out that only a RL flat fuzzy number \tilde{A}_2 can be subtracted from an LR flat fuzzy number \tilde{A}_1 i.e., if \tilde{A}_1 and \tilde{A}_2 both are LR flat fuzzy numbers such that $L(\cdot) \neq R(\cdot)$ then $\tilde{A}_1 \ominus \tilde{A}_2$ does not exist. Hence, if all the parameters of the fully fuzzy linear programming problem (P_2) are represented by such LR flat fuzzy numbers for which $L(\cdot) \neq R(\cdot)$ then due to the existence of $\sum_{j \in N_1} \tilde{c}_j \otimes \tilde{x}_j \ominus \sum_{j \in N_2} \tilde{c}_j \otimes \tilde{x}_j$ and $\sum_{j \in N_3} \tilde{a}_{ij} \otimes \tilde{x}_j \ominus \sum_{j \in N_4} \tilde{a}_{ij} \otimes \tilde{x}_j$, the fully fuzzy linear programming problem (P_2) is not valid.

Example 3.1 A manufacturer of biscuits is considering four types of gift packs containing three types of biscuits: orange cream (OC), chocolate cream (CC) and wafers (W). A market research conducted recently according to consumer preferences demonstrated that the assortments shown in Table 1 are to be in demand.

For the biscuits, the fuzzy manufacturing capacity and fuzzy costs are shown in Table 2.

Formulate a model to find the production schedule which maximizes the fuzzy profit by assuming that there are no market restrictions.

| Assortments | Contents | Fuzzy selling price per kg. (Rs.) |
|-------------|---|-----------------------------------|
| A | Not less than 40% of OC, not more than 20% of CC, any quantity of W | $(10,30,10,10)_{LR}$ |
| B | Not less than 20% of OC, not more than 40% of CC, any | $(20,30,10,10)_{LR}$ |

| | | |
|---|---|----------------------|
| | quantity of W | |
| C | Not less than 50% of OC, not more than 10% of CC, any quantity of W | $(22,22,12,12)_{LR}$ |
| D | No restrictions | $(10,14,2,2)_{LR}$ |

Table 1. Most preferred assortments of biscuits.

| Biscuit variety | OC | CC | W |
|-----------------------------------|-------------------------|-------------------------|-------------------------|
| Fuzzy plant capacity (kg/day) | $(150,250, 50,50)_{LR}$ | $(180,220, 20,20)_{LR}$ | $(100,200, 50,50)_{LR}$ |
| Fuzzy manufacturing cost (Rs./kg) | $(8,8,8,8)_{LR}$ | $(9,10,4,2)_{LR}$ | $(5,9,5,5)_{LR}$ |

Table 2. Fuzzy manufacturing capacity and fuzzy cost for biscuits.

3.1 Existing fuzzy linear programming formulation of the chosen problem

Using the existing general form of fully fuzzy linear programming problems (P_2) , the chosen problem can be formulated into the following fully fuzzy linear programming problem

Maximize

$$(10,30,10,10)_{LR} \otimes (\tilde{x}_{A1} \oplus \tilde{x}_{A2} \oplus \tilde{x}_{A3}) \oplus (20,30,10,10) \otimes (\tilde{x}_{B1} \oplus \tilde{x}_{B2} \oplus \tilde{x}_{B3}) \oplus (22,22,12,12) \otimes (\tilde{x}_{C1} \oplus \tilde{x}_{C2} \oplus \tilde{x}_{C3}) \oplus (10,14,2,2) \otimes (\tilde{x}_{D1} \oplus \tilde{x}_{D2} \oplus \tilde{x}_{D3}) \ominus (8,8,8,8) \otimes (\tilde{x}_{A1} \oplus \tilde{x}_{B1} \oplus \tilde{x}_{C1} \oplus \tilde{x}_{D1}) \ominus (9,10,4,2) \otimes (\tilde{x}_{A2} \oplus \tilde{x}_{B2} \oplus \tilde{x}_{C2} \oplus \tilde{x}_{D2}) \ominus (5,9,5,5) \otimes (\tilde{x}_{A3} \oplus \tilde{x}_{B3} \oplus \tilde{x}_{C3} \oplus \tilde{x}_{D3})$$

Subject to

$$\begin{aligned} \tilde{x}_{A1} &\geq_{\mathfrak{R}} 0.40(\tilde{x}_{A1} \oplus \tilde{x}_{A2} \oplus \tilde{x}_{A3}) \\ \tilde{x}_{B1} &\geq_{\mathfrak{R}} 0.20(\tilde{x}_{B1} \oplus \tilde{x}_{B2} \oplus \tilde{x}_{B3}) \\ \tilde{x}_{C1} &\geq_{\mathfrak{R}} 0.50(\tilde{x}_{C1} \oplus \tilde{x}_{C2} \oplus \tilde{x}_{C3}) \\ \tilde{x}_{A2} &\leq_{\mathfrak{R}} 0.20(\tilde{x}_{A1} \oplus \tilde{x}_{A2} \oplus \tilde{x}_{A3}) \\ \tilde{x}_{B2} &\leq_{\mathfrak{R}} 0.40(\tilde{x}_{B1} \oplus \tilde{x}_{B2} \oplus \tilde{x}_{B3}) \\ \tilde{x}_{C2} &\leq_{\mathfrak{R}} 0.10(\tilde{x}_{C1} \oplus \tilde{x}_{C2} \oplus \tilde{x}_{C3}) \\ \tilde{x}_{A1} \oplus \tilde{x}_{B1} \oplus \tilde{x}_{C1} \oplus \tilde{x}_{D1} &\leq_{\mathfrak{R}} (150,250,50,50) \end{aligned} \quad (P_3)$$

$$\tilde{x}_{A2} \oplus \tilde{x}_{B2} \oplus \tilde{x}_{C2} \oplus \tilde{x}_{D2} \leq_{\mathfrak{R}} (180, 220, 20, 20)$$

$$\tilde{x}_{A3} \oplus \tilde{x}_{B3} \oplus \tilde{x}_{C3} \oplus \tilde{x}_{D3} \leq_{\mathfrak{R}} (100, 200, 50, 50)$$

Where, \tilde{x}_{ij} ($i = A, B, C, D; j = 1, 2, 3$) is a non-negative LR flat fuzzy number and

(i) $\tilde{x}_{A1}, \tilde{x}_{A2}, \tilde{x}_{A3}$ denote the fuzzy quantity in kg. of OC, CC and W type of biscuits for the gift pack A.

(ii) $\tilde{x}_{B1}, \tilde{x}_{B2}, \tilde{x}_{B3}$ denote the fuzzy quantity in kg. of OC, CC and W type of biscuits for the gift pack B.

(iii) $\tilde{x}_{C1}, \tilde{x}_{C2}, \tilde{x}_{C3}$ denote the fuzzy quantity in kg. of OC, CC and W type of biscuits for the gift pack C.

(iv) $\tilde{x}_{D1}, \tilde{x}_{D2}, \tilde{x}_{D3}$ denote the fuzzy quantity in kg. of OC, CC and W type of biscuits for the gift pack D.

3.2 Drawbacks of the obtained fuzzy optimal value

To the best of our knowledge, only the existing method [1] can be used to find the fuzzy optimal value of such fully fuzzy linear programming problems in which the parameters are represented by LR flat fuzzy numbers.

In this section, to show the shortcomings of the existing general form of fully fuzzy linear programming problems (P_2), the fuzzy optimal value of fully fuzzy linear programming problem (P_3) obtained by using the existing method [1] is shown in Table 3.

| Method | Fuzzy optimal value | $L(\cdot)$ & $R(\cdot)$ |
|---------------------|----------------------|--|
| Existing method [1] | (P_3) is not valid | $L(x) = \text{maximum}\{0, 1 - x\}$ $R(x) = \text{maximum}\{0, 1 - x^2\}$ |
| Existing method [1] | (P_3) is not valid | $L(x) = \text{maximum}\{0, 1 - x\}$ $R(x) = \text{maximum}\{0, 1 - x^4\}$ |

Table 3. Results of the chosen problem using existing formulation (P_3).

In the objective function of fully fuzzy linear programming formulation (P_3) of the problem, chosen in Example 3.1, subtraction of two LR flat fuzzy numbers is occurring. Hence, if $L(\cdot) \neq R(\cdot)$ then the fully fuzzy linear programming formulation (P_3) is not valid i.e., it is not possible to find the fuzzy optimal solution of the chosen problem by using the fully fuzzy linear programming formulation (P_3).

Remark 1. The shortcomings, pointed out in Section 3, will also occur in the existing general form of fuzzy linear programming problems [5-11, 18-30].

4. Proposed general form of fully fuzzy linear programming problems

In this section, to solve the shortcomings of the existing general form of fully fuzzy linear programming problems, pointed out in Section 3, a new general form of the fully fuzzy linear programming problems is proposed.

Maximize/Minimize S

Subject to

$$\sum_{j \in N_1} c_j x_j = S + \sum_{j \in N_2} c_j x_j \quad (P_4)$$

$$\sum_{j \in N_3} a_{ij} x_j \leq, =, \geq b_i + \sum_{j \in N_4} a_{ij} x_j, \quad i = 1, 2, \dots, m$$

Where, x_j , a_{ij} , b_i , c_j and S are real numbers.

Replacing all the crisp parameters of the crisp linear programming problems (P_4) by fuzzy parameters general form of the fully fuzzy linear programming problems (P_5) can be written as

Maximize/Minimize \tilde{S}

Subject to

$$\sum_{j \in N_1} \tilde{c}_j \otimes \tilde{x}_j = \tilde{S} \oplus \sum_{j \in N_2} \tilde{c}_j \otimes \tilde{x}_j \quad (P_5)$$

$$\sum_{j \in N_3} a_{ij} \otimes x_j \leq_{\mathfrak{R}}, =_{\mathfrak{R}}, \geq_{\mathfrak{R}} \tilde{b}_i \oplus \sum_{j \in N_4} \tilde{a}_{ij} \otimes \tilde{x}_j,$$

$$i = 1, 2, \dots, m$$

Where, \tilde{x}_j , \tilde{a}_{ij} , \tilde{b}_i , \tilde{c}_j and \tilde{S} are LR flat fuzzy numbers.

4.1 Advantages of the proposed general form of fully fuzzy linear programming problems

Because in the proposed general form (P_5) the subtraction of LR flat fuzzy numbers is not occurring, hence using the proposed general form the shortcomings of the existing general form pointed out in Section 3 are solved.

To show the advantage of the proposed general form over the existing general form, it was demonstrated that if the problem chosen in Example 3.1 is formulated by using the proposed general form, then all the shortcomings pointed out in Section 3 are solved.

Using the proposed general form of fully fuzzy linear programming problems (P_5), the problem, chosen in Example 3.1, can be formulated into fully fuzzy linear programming problem (P_6):

Maximize \tilde{P}

Subject to

$$(10,30,10,10)_{LR} \otimes (\tilde{x}_{A1} \oplus \tilde{x}_{A2} \oplus \tilde{x}_{A3}) \oplus (20,30,10,10) \otimes (\tilde{x}_{B1} \oplus \tilde{x}_{B2} \oplus \tilde{x}_{B3}) \oplus (22,22,12,12) \otimes (\tilde{x}_{C1} \oplus \tilde{x}_{C2} \oplus \tilde{x}_{C3}) \oplus (10,14,2,2) \otimes (\tilde{x}_{D1} \oplus \tilde{x}_{D2} \oplus \tilde{x}_{D3}) = \tilde{P}$$

$$\oplus (8,8,8,8) \otimes (\tilde{x}_{A1} \oplus \tilde{x}_{B1} \oplus \tilde{x}_{C1} \oplus \tilde{x}_{D1}) \oplus (9,10,4,2) \otimes (\tilde{x}_{A2} \oplus \tilde{x}_{B2} \oplus \tilde{x}_{C2} \oplus \tilde{x}_{D2}) \oplus (5,9,5,5) \otimes (\tilde{x}_{A3} \oplus \tilde{x}_{B3} \oplus \tilde{x}_{C3} \oplus \tilde{x}_{D3})$$

$$\tilde{x}_{A1} \geq_{\mathfrak{R}} 0.40(\tilde{x}_{A1} \oplus \tilde{x}_{A2} \oplus \tilde{x}_{A3})$$

$$\tilde{x}_{B1} \geq_{\mathfrak{R}} 0.20(\tilde{x}_{B1} \oplus \tilde{x}_{B2} \oplus \tilde{x}_{B3})$$

$$\tilde{x}_{C1} \geq_{\mathfrak{R}} 0.50(\tilde{x}_{C1} \oplus \tilde{x}_{C2} \oplus \tilde{x}_{C3}) \quad (P_6)$$

$$\tilde{x}_{A2} \leq_{\mathfrak{R}} 0.20(\tilde{x}_{A1} \oplus \tilde{x}_{A2} \oplus \tilde{x}_{A3})$$

$$\tilde{x}_{B2} \leq_{\mathfrak{R}} 0.40(\tilde{x}_{B1} \oplus \tilde{x}_{B2} \oplus \tilde{x}_{B3})$$

$$\tilde{x}_{C2} \leq_{\mathfrak{R}} 0.10(\tilde{x}_{C1} \oplus \tilde{x}_{C2} \oplus \tilde{x}_{C3})$$

$$\tilde{x}_{A1} \oplus \tilde{x}_{B1} \oplus \tilde{x}_{C1} \oplus \tilde{x}_{D1} \leq_{\mathfrak{R}} (150,250,50,50)$$

$$\tilde{x}_{A2} \oplus \tilde{x}_{B2} \oplus \tilde{x}_{C2} \oplus \tilde{x}_{D2} \leq_{\mathfrak{R}} (180,220,20,20)$$

$$\tilde{x}_{A3} \oplus \tilde{x}_{B3} \oplus \tilde{x}_{C3} \oplus \tilde{x}_{D3} \leq_{\mathfrak{R}} (100,200,50,50)$$

Where, \tilde{x}_{ij} ($i = A, B, C, D; j = 1, 2, 3$) is a non-negative LR flat fuzzy number and \tilde{P} is an LR flat fuzzy number.

Fuzzy optimal value of the formulated problem (P_6) by using the existing method [1] is shown in Table 4.

| Method | Fuzzy optimal value | $L(\cdot)$ & $R(\cdot)$ |
|---------------------|----------------------------------|--|
| Existing method [1] | (14555,14555, 0,0) _{LR} | $L(x) = \text{maximum}\{0, 1 - x\}$ $R(x) = \text{maximum}\{0, 1 - x^2\}$ |
| Existing method [1] | (14759,14759, 0,0) _{LR} | $L(x) = \text{maximum}\{0, 1 - x\}$ $R(x) = \text{maximum}\{0, 1 - x^4\}$ |

Table 4. Results of the chosen problem using proposed formulation (P_6).

It is obvious from Table 4 that the proposed formulation (P_6) is valid for all values of $L(\cdot)$ and $R(\cdot)$. Therefore, by using the proposed general form of fully fuzzy linear programming problems (P_5) all the shortcomings, pointed out in Section 3, are solved.

5. Conclusions

Based on the present study it can be concluded that it is better to use the proposed general form of fully fuzzy linear programming problems as it was compared to the existing general form of fully fuzzy linear programming problems.

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