



A novel quadriphase ZCZ sequences for QS-CDMA systems

B. Fassi^a • S. Driz^{a*} • I. Merrabi^b

^aThe University of Djillali Liabes, Telecommunications and Digital
Signal Processing Laboratory, Sidi Bel Abbes, Algeria

^bSidi Bel Abbes, Algeria

Received 10 10 2023; accepted 04 09 2024

Available 06 30 2024

Abstract: In this article, a novel method for constructing quadriphase Zero Correlation Zone (ZCZ) sequence sets based on the mutually orthogonal complementary sets (MOCS) matrix and polyphase perfect sequence is presented. Based on the obtained MATLAB results in terms of correlation functions, it can be mentioned that the resultant ZCZ set is suitable for quasi-synchronous Code Division Multiple Access (QS-CDMA) systems. The obtained sequence sets are near-optimal concerning mathematical bound, and their construction is more flexible than other polyphase ZCZ constructions. Furthermore, their parameters can be chosen flexibly.

Keywords: MOCS matrix, perfect sequence, QS-CDMA, ZCZ codes

*Corresponding author.

E-mail address: samia7922@yahoo.fr (S. Driz).

Peer Review under the responsibility of Universidad Nacional Autónoma de México.

1. Introduction

Code Division Multiple Access (CDMA) is a promising channel access method allowing a multiuser communication scheme where multiplexed users simultaneously share the same access network. In such systems, unique signature sequences are dedicated to different users to ensure secure network connection, flexibility, and multimedia services. Several CDMA categories have been proposed such as synchronous (Abd et al., 2012; Driz & Djebbari, 2019; Driz et al., 2023; Mrabet et al., 2020), asynchronous (Liu et al., 2024), and quasi-synchronous (QS) architectures (Fassi & Driz, 2023; Kumar et al., 2023) applied to different network scenarios. Accordingly, different coding schemes with adapted correlation properties have been developed. In a synchronous OCDMA system where no relative time delay between the user's signals is considered ($\tau = 0$), spreading sequences with zero cross correlation (ZCC) properties are required to avoid multiple access interference (MAI) (Driz et al., 2022; Li et al., 2020; Sarangal et al., 2021). Nevertheless, in some situations where $\tau \neq 0$ (asynchronous systems), code sequences with Zero Correlation Zone (ZCZ) properties are needed to eliminate the MAI effects (Fassi & Taleb-Ahmed, 2018; Wang et al., 2024).

Another CDMA system (quasi-synchronous architecture) is defined when $\tau \neq 0$ in some limited time range, smaller than the sequence length. In such architecture, the ZCZ codes are the suitable choice where the maximum time delay is well known as the ZCZ zone (Chen et al., 2019; Driz & Fassi, 2022). On the other hand, several ZCZ code constructions such as binary, ternary, polyphase, and optical codes have been proposed. Binary ZCZ codes, characterized by binary symbols typically denoted as -1 and 1, face challenges in simultaneously increasing:

- Sequence length L improves noise immunity but reduces data rate and potentially increases complexity,
- Set size M allows for more users but might require reduced ZCZ length,
- Zero-correlation length ZCZ: reduces interference but comes at the cost of potentially longer sequences.

These parameters are all interconnected, and there is a fundamental trade-off to be made between these factors depending on the specific needs of the communication system. Thus, constructing optimal ZCZ sequences tailored for communication systems necessitates specific criteria - mathematical bounds- to help determine the theoretical limits for achieving good ZCZ properties with a given sequence length effects (Fassi & Taleb-Ahmed 2018). In contrast, ternary ZCZ codes, utilizing three levels usually labeled as -1, 0, and 1, were developed to address this challenge by enhancing the set size to reach a more relaxed upper bound in terms of the sequence set compared to their binary counterparts. The theoretical upper bound, contingent upon the phase shift's

absolute value being less than or equal to ZCZ width, is expressed as $M \cdot (Zcz + 1) \leq L$, called the Tang-Fan-Matsufuji bound (Donelan & O'Farrell, 2002; Fassi et al., 2014; Hayashi et al., 2018; Matsufuji, et al., 2015).

Meanwhile, polyphase ZCZ codes incorporate complex symbols, expanding beyond simple binary or ternary representations. Polyphase ZCZ codes introduce a rich tapestry of symbols, including not only 1 and -1 but also 0, j , $-j$, and potentially an array of other complex elements. This expansion into the domain of complex symbols endows polyphase ZCZ codes with unprecedented versatility and adaptability. One of the standout features of polyphase ZCZ codes is their inherent capacity to accommodate expanding sets of codes with varying sizes. This means that instead of being confined to fixed-length sequences, polyphase ZCZ codes can seamlessly adapt to generate code sets of diverse lengths. This flexibility is instrumental in addressing the multifaceted demands of modern communication systems, where requirements can vary widely depending on factors such as channel conditions, bandwidth constraints, and signal processing capabilities (Chen, et al., 2021; Li et al., 2019; Torii et al., 2013). Moreover, Polyphase ZCZ codes have the property that their maximum autocorrelation values are typically equal to the length of the sequence at a specific shift – shift 0. This characteristic is desirable in code design as it helps minimize interference between different sequences. Furthermore, the incorporation of polyphase sequences enhances the security level of the system, as their complex construction methods make interception difficult. This reinforces their suitability for robust communication applications.

On the other hand, optical ZCZ (OZCZ) has been adapted and applied to optical communication to encode data onto optical signals, typically in the form of pulses or modulated light. This adaptation serves to mitigate interference and bolster signal integrity, rendering OZCZ a pivotal strategy for optimizing the efficacy of quasi-synchronous optical communication systems (Chen et al., 2022; Driz et al., 2023; Ohira et al., 2022; Ouis et al., 2022).

In this paper, we propose a novel method for constructing quadriphase sets of ZCZ complex spreading sequences based on the MOCS matrix and polyphase perfect sequence to achieve correlation properties suited for QS-CDMA applications.

The remainder of this paper is organized as follows. Section 2 gives some definitions and nomenclature required for the rest of the document. Sets of sequences with desired correlation properties are constructed by using the proposed method in Section 3. Section 4 provides an example of a quadriphase ZCZ spreading sequence followed by the properties of the proposed code in Section 5. An examination of the system's performance concerning Bit Error Rate (BER) is detailed in Section 6. Conclusions concludes the study.

2. Preliminary considerations

2.1. Periodic correlation function

The periodic correlation function (PCF), $\theta(b_i, b_{i'})(\tau)$ between a pair of binary sequences of length L at a lag τ is defined by Driz et al. (2023):

$$\forall \tau \geq 0, \quad \begin{cases} \theta(b_i, b_{i'})(\tau) = \sum_{j=0}^{L-1-\tau} b_{i,j} b_{i',(j+\tau) \bmod L} \\ \theta(b_i, b_{i'})(-\tau) = \theta(b_i, b_{i'})(\tau) \end{cases} \quad (1)$$

2.2. Aperiodic correlation function

The aperiodic correlation function (ACF), $\varphi(b_i, b_{i'})(\tau)$ for general polyphase sequences of length L is defined as follows (Li et al., 2019; Wysocki, 2003):

$$\varphi(b_i, b_{i'})(\tau) = \begin{cases} \frac{1}{L} \sum_{j=0}^{L-\tau-1} b_{i,j} b_{i',(j+\tau)}^* & 0 \leq \tau \leq L-1 \\ \frac{1}{L} \sum_{j=0}^{L+\tau-1} b_{i,(j-\tau)} b_{i',j}^* & 1-L \leq \tau < 0 \\ 0 & |\tau| \geq L \end{cases} \quad (2)$$

where $[\cdot]^*$ denotes a complex conjugate operation.

2.3. ZCZ sequences

A set of M sequences is called an optical Zero Correlation Zone (ZCZ) sequence set, denoted by $ZCZ(L, M, Z_{CZ})$, where $Z_{CZ} = \min\{Z_a, Z_c\}$ indicate the Zero Correlation Zone if the periodic correlation function satisfies (Driz et al., 2023):

$$\theta(b_i, b_{i'})(\tau) = \begin{cases} \omega ; \tau = 0, i = i' \\ 0 ; i = i', 1 \leq |\tau| \leq Z_a \\ 0 ; i \neq i', |\tau| \leq Z_c \end{cases} \quad (3)$$

2.4. Perfect sequences

A sequence $b = (b_0, b_1, \dots, b_{L_1-1})$ of length L_1 is said to be a perfect sequence if the ideal periodic autocorrelation function (PACF) is given by (Hayashi, 2007; Hayashi & Matsufuji, 2009):

$$\theta_{b,b}(\tau) = \sum_{l=0}^{L_1-1} b_l b_{l+\tau} = \begin{cases} \eta L_1, \eta \leq 1, & \text{for } \tau = 0 \\ 0, & \text{for } 1 \leq \tau \leq L_1 - 1 \end{cases} \quad (4)$$

where η denotes energy efficiency.

3. Quadriphase ZCZ code construction

The construction procedure of the proposed quadriphase OZCZ codes is performed in three steps.

3.1. Step 1

First, let $F^{(n)}$ ($n \geq 0$) denote a MOCS matrix of order $(M \times M) = (2^{n+1} \times 2^{n+1})$ where each element of $F^{(n)}$ is a sequence of length $l_e = 2^n * l_m = 2^{m+n}$ ($l_m = 2^m, m \geq 0$) and each line have a length of $L = 2^{2n+m+1}$. Thus, an MOCS matrix of $(2^{n+1} \times 2^{2n+m+1})$ where $n \geq 0$ and $m \geq 0$ is arranged as follows (Fan et al., 1999):

$$F^{(n)} = \begin{bmatrix} F_{11} & F_{12} & \dots & F_{1k} & F_{1M} \\ F_{21} & F_{22} & \dots & F_{2k} & F_{2M} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ F_{i1} & F_{i2} & \dots & F_{ik} & F_{iM} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ F_{M1} & F_{M2} & \dots & F_{Mk} & F_{MM} \end{bmatrix}_{(M \times M)} = \begin{bmatrix} f_0 \\ f_1 \\ \vdots \\ f_i \\ \vdots \\ f_{M-1} \end{bmatrix} \quad (5)$$

Further, $F^{(n)}$ can be considered as a MOCS matrix when it satisfies the following formulas (Tseng & Liu, 1972):

$$\sum_{k=1}^M \varphi(F_{i,k}, F_{i,k})(\tau) = 0, \quad \forall i, \forall \tau \neq 0 \quad (6)$$

$$\sum_{k=1}^M \varphi(F_{i,k}, F_{i',k})(\tau) = 0, \quad \forall i \neq i', \forall \tau. \quad (7)$$

Where $\varphi(F_{i,k}, F_{i,k})(\tau)$ and $\varphi(F_{i,k}, F_{i',k})(\tau)$ are the aperiodic autocorrelation and aperiodic cross-correlation functions, respectively.

3.2. Step 2

Let $A_i = (a_0^i, a_1^i, \dots, a_s^i, \dots, a_{l_1-1}^i)$ denote a perfect sequence of length l (l must be an even integer and $0 \leq i \leq l_1 - 1$) whose elements are complex numbers having absolute value 1. Let l_0 and l_1 be two integers such that $l = l_1 * l_0$ ($1 \leq l_1 < l$ and $1 \leq l_0 < l$).

By using the elementary transformation for perfect sequences that shifts the sequence cyclically to the left by $(i * l_0)$, we can generate more than one perfect sequence. The set of l_1 generated perfect sequences is denoted by A :

$$A = \begin{bmatrix} A_0 \\ A_1 \\ \vdots \\ A_i \\ \vdots \\ A_{l_1-1} \end{bmatrix} = \begin{bmatrix} a_0^0 & a_1^0 & \dots & a_s^0 & \dots & a_{l_1-1}^0 \\ a_0^1 & a_1^1 & \dots & a_s^1 & \dots & a_{l_1-1}^1 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ a_0^i & a_1^i & \dots & a_s^i & \dots & a_{l_1-1}^i \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ a_0^{l_1-1} & a_1^{l_1-1} & \dots & a_s^{l_1-1} & \dots & a_{l_1-1}^{l_1-1} \end{bmatrix} = [b_0 \ b_1 \ \dots \ b_s \ \dots \ b_{l_1-1}] \quad (8)$$

$$\text{where: } b_s = \begin{bmatrix} a_s^0 \\ a_s^1 \\ \vdots \\ a_s^i \\ \vdots \\ a_s^{l_1-1} \end{bmatrix}$$

next, we convert the matrix A into a single-column matrix denoted by B as follows:

$$B = \begin{bmatrix} b_0 \\ b_1 \\ \vdots \\ b_s \\ \vdots \\ b_{l_1-1} \end{bmatrix} \tag{9}$$

The order of elements in the matrix B is by column. Indeed, the values are taken column by column from the matrix A by moving down one column and then across to the right to the next column. The obtained B matrix contains $(l * l_1) - 1$ rows.

3.3. Step 3

Using the elements of the matrix B , we generate the matrix Q_r of length $(l * l_1 * l_e)$ by concatenating each element in the B matrix l_e times, as follows:

For $0 \leq r \leq (l * l_1) - 1$:

$$Q_r = [B(r) \cdots B(r)] |_{l_e \text{ times}} \tag{10}$$

where $B(r)$ denotes the r^{th} element of the matrix B . Next, we convert matrix Q_r to one dimensional array matrix denoted Q of length $(l_e * l_1 * l)$ by concatenating horizontally all obtained rows of the matrix Q_r as follows:

$$Q = [Q_0 Q_1 Q_2 \dots \dots \dots Q_{(l * l_1) - 2} Q_{(l * l_1) - 1}] \tag{11}$$

On the other hand, let us define D_i sequence obtained from the MOCS matrix $F^{(n)}$ by concatenating its i^{th} arrays (f_i) horizontally l times as follows:

For $0 \leq i \leq l_1 - 1$:

$$D_i = [(f_i f_i \cdots f_i)] |_{l \text{ times}} \tag{12}$$

Finally, the obtained quaternary set C contains C_i sequences, each of length $(l * L)$, given by :

$$C = \begin{bmatrix} C_0 \\ \vdots \\ C_i \\ \vdots \\ C_{l_1-1} \end{bmatrix} = \begin{bmatrix} c_0^0 & \dots & c_0^j & \dots & c_0^{(l * L) - 1} \\ & \vdots & & \ddots & \vdots \\ c_i^0 & \dots & c_i^j & \dots & c_i^{(l * L) - 1} \\ & \vdots & & \ddots & \vdots \\ c_{l_1-1}^0 & \dots & c_{l_1-1}^j & \dots & c_{l_1-1}^{(l * L) - 1} \end{bmatrix} \tag{13}$$

where $0 \leq i \leq l_1 - 1$ and $0 \leq j \leq (l * L) - 1$.

The quadriphase sequences can be derived from Equations 11 and 12 as follows:

$$C_i = D_i \otimes Q \tag{14}$$

$$c_i^j = f_{i, j \bmod L} \cdot Q_j \tag{15}$$

where \otimes denotes the element-by-element multiplication and c_i^j is the j^{th} element of C_i .

4. Example of quadriphase ZCZ spreading sequences

4.1. Step 1

We choose $F^{(1)}$ a MOCS matrix of $(M \times M) = (4 \times 4)$ (with $n = 1$ and $m = 0$), obtained by using Equation 5 as follows:

$$F^{(1)} = \begin{bmatrix} + & + & - & - & - & + & + & - \\ + & + & + & + & - & + & - & + \\ - & + & + & - & + & + & - & - \\ - & + & - & + & + & + & + & + \end{bmatrix} = \begin{bmatrix} f_0 \\ f_1 \\ f_2 \\ f_3 \end{bmatrix}$$

where $(-)$ and $(+)$ denote (-1) and $(+1)$, respectively. In this case, $l_e = 2^{m+n} = 2$ and $L = 2^{2n+m+1} = 8$.

4.2. Step 2

In this step, we choose a perfect sequence, denoted by A_0 , of length $l = 8$ ($l_1 = 4$ and $l_0 = 2$) given by: $A_0 = (1, 1, j, 1, 1, -1, j, -1)$. For $0 \leq i \leq 3$, we can construct four (04) perfect sequences of length $l = 8$ as follows:

$$A_0 = (00100212) , A_1 = (10021200),$$

$$A_2 = (02120010) , A_3 = (12001002).$$

Where $0, 1, \text{ and } 2$ denote $1, j, -1$ respectively and $j = \sqrt{-1}$.

From the obtained sequences, we apply Equation 8 to find the matrix A (4×8) as follows:

$$A = \begin{bmatrix} A_0 \\ A_1 \\ A_2 \\ A_3 \end{bmatrix} = \begin{bmatrix} 00100212 \\ 10021200 \\ 02120010 \\ 12001002 \end{bmatrix} = [b_0 b_1 b_2 b_3 b_4 b_5 b_6 b_7]$$

Where:

$$b_0 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}; b_1 = \begin{bmatrix} 0 \\ 0 \\ 2 \\ 2 \end{bmatrix}; b_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}; \dots \dots \dots; b_6 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}; b_7 = \begin{bmatrix} 2 \\ 0 \\ 0 \\ 2 \end{bmatrix}$$

Similarly, the matrix $B (32 \times 1)$ is obtained from Equation 9 as follows:

$$B = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 2 \\ 2 \\ \vdots \\ 1 \\ 0 \\ 1 \\ 0 \\ 2 \\ 0 \\ 0 \\ 2 \end{bmatrix}$$

4.3. Step 3

For $0 \leq r \leq 31$, we use Equations 10 and 11 to find the matrix $Q_r (32 \times 2)$ and $Q (64 \times 1)$ as given below:

$$Q_r = \begin{bmatrix} 0 & 0 \\ 1 & 1 \\ 0 & 0 \\ 1 & 1 \\ 0 & 0 \\ 0 & 0 \\ 2 & 2 \\ 2 & 2 \\ \vdots & \vdots \\ 1 & 1 \\ 0 & 0 \\ 1 & 1 \\ 0 & 0 \\ 2 & 2 \\ 0 & 0 \\ 0 & 0 \\ 2 & 2 \end{bmatrix},$$

where:

$$Q_0 = [00]; Q_1 = [11], Q_2 = [00], \dots, Q_{30} = [00], Q_{31} = [22] \text{ and } Q = [0011001100002222110011000022220000110011222200001100110022000022].$$

For $0 \leq i \leq 3$ and using Equation 12, the sequences D_i are given by:

$$\begin{bmatrix} D_0 \\ D_1 \\ D_2 \\ D_3 \end{bmatrix} = \begin{bmatrix} + & + & - & - & + & - & \dots & + & + & - & - & + & - & \dots & + & + & - & - & + & - \\ + & + & + & - & - & - & \dots & + & + & + & - & - & - & \dots & + & + & + & - & - & - \\ - & + & + & - & + & - & \dots & + & + & - & + & - & \dots & + & + & - & + & - & \dots & + \\ - & + & - & + & + & + & \dots & - & + & - & + & + & \dots & - & + & - & + & + & \dots & - \end{bmatrix}$$

Finally, we obtain four (04) quadriphase sequences of $ZCZ(N, M, Z_{cz}) = ZCZ(64, 4, 12)$:

$$C_0 = (0033201300220220112231020000021200332013220020021122310222222021)$$

$$C_1 = (001120310000020211003120002202200011203122220201100212022002002)$$

$$C_2 = (2013003320022200310211222020222220130033022000223102112202020000)$$

$$C_3 = (2031001120202222312011002002220020310011020200003120110002200022)$$

where $0, 1, 2, 3$ denote $1, j, -1, -j$ respectively and $j = \sqrt{-1}$.

The correlation functions (PACF and PCCF periodic cross-correlation function) of the obtained sequences are illustrated in Figure 1 and Figure 2.

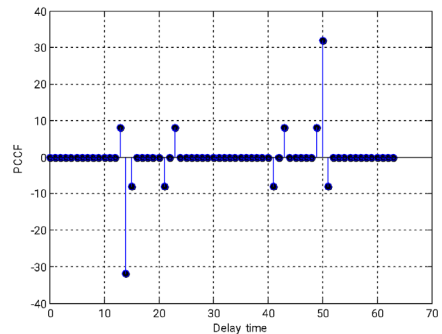


Figure 1. PACF of C_0 .

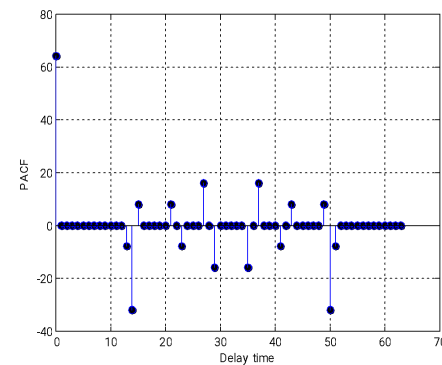


Figure 2. PCCF of C_0 with C_1 .

5. Properties of the proposed code

The proposed quadriphase ZCZ code can be generated from the MOCS matrix and polyphase perfect sequence. The generated sequence set satisfies the following properties:

$$\left\{ \begin{array}{l} \forall i, \forall \tau \neq 0, |\tau| \leq (l - 2)2^{n+m} \\ \theta(C_i, C_i)(\tau) = 0 \end{array} \right.$$

and

$$\left\{ \begin{array}{l} \forall i \neq v, \forall \tau, |\tau| \leq (l - 2)2^{n+m} \\ \theta(C_i, C_v)(\tau) = 0 \end{array} \right.$$

{C} is the set of ZCZ sequences with parameters $(N, M, Z_{CZ}) = (l(2^{2n+m+1}), 2^{n+1}, (l - 2)2^{n+m})$. The parameters of the proposed construction satisfy the theory of the bound of ZCZ codes. Let be the ratio of optimality:

$$\begin{aligned} \rho &= \frac{M(Z_{CZ} + 1)}{N} = \frac{2^{n+1}[(l - 2)2^{n+m} + 1]}{l(2^{2n+m+1})} \\ &= 1 - \frac{2}{l} + \frac{1}{l(2^{n+m})} < 1. \end{aligned}$$

Table 1 gives the bound of the proposed construction according to the parameters (n, m) of the MOCS matrix and the length l of perfect sequence. From Table 1, we note that when the length of the perfect sequence increases, the optimal ratio of proposed sequences approaches the theoretical limit of ZCZ sequences.

Table 1. Bound of the proposed ZCZ codes.

l		2	8	16	32
ρ	n + m = 0	0.50	0.87	0.93	0.96
	n + m ≠ 0 (n = 1 and m = 0) (n = 0 and m = 1)	0.25	0.81	0.90	0.95
	n + m > 0 (n = 1 and m = 1)	0.12	0.78	0.89	0.94

Compared with the construction of polyphase sequences of TORII (Torii, 2001), the set of the proposed sequences has several advantages. First, the principal advantage of this construction method is that we can construct a set of ZCZ quaternary sequences based on the MOCS matrix, whose elements are (+1) and (-1), instead of the polyphase unitary matrix whose elements are complex numbers. Furthermore, the use of the MOCS matrix in the construction gives more flexibility suitable to define the desired parameters (the length L, the capacity M, and the zone ZCZ) to cope with MAI and ISI

interferences in the channel. On the other hand, an improvement in the ZCZ parameter is due to the (n,m) parameters of the MOCS matrix.

Example: for l = 16 and (n, m) variables, we can notice:

- For MOCS (1, 0), the proposed set is of parameters ZCZ (128, 4, 28) and ρ = 0.90 .
- For MOCS (1, 1), the proposed set is of parameters ZCZ (256, 4, 56) and ρ = 0.89 .

- For MOCS (2, 0), the proposed set is of parameters ZCZ (512, 8, 56) and ρ = 0.89 .

- For MOCS (2, 1), the proposed set is of parameters ZCZ (1024, 8, 112) and ρ = 0.88.

- For MOCS (2, 2), the proposed set is of parameters ZCZ (2048, 8, 224) and ρ = 0.87.

- For MOCS (3, 0), the proposed set is of parameters ZCZ (2048, 16, 112) and ρ = 0.89.

6. System analysis

In DS CDMA systems, minimizing the maximum values of aperiodic cross-correlation functions and out-of-phase aperiodic autocorrelation functions is required. Pursley (1977) demonstrated that for a BPSK asynchronous DS CDMA system, the average signal-to-noise ratio (SNR) at the receiver output of the ith user's correlator receiver can be formulated in terms of the average interference parameter (AIP) from the other M users in the system and the power of white Gaussian noise within the channel.

The SNR for ith desired user denoted as SNR (i), can be expressed in the form:

$$SNR(i) = \left[\frac{N_0}{2E_b} + \frac{1}{6L^3} \left(\sum_{m=1}^M \sum_{m \neq i} r_{m,i} \right) \right]^{-0.5} \tag{16}$$

where E_b is the bit energy, N₀ is one-sided Gaussian noise power spectral density, and r_{m,i} is the average interference parameter (AIP), defined for a pair of sequences as:

$$r_{m,i} = 2 \cdot \mu_{m,i}(0) + Re\{\mu_{m,i}(1)\}. \tag{17}$$

The cross-correlation parameters μ_{m,i}(τ) are defined as:

$$r_{m,i} = L^2 \sum_{l=1}^{L-1-L} C_{m,i}(l) [C_{m,i}(l + \tau)]^* \tag{18}$$

where C_{m,i}(l) = φ_(b_m, b_i)(l) represents the aperiodic correlation function (ACF), for general polyphase sequences of length L, defined in Equation 2.

However, following the derivation in Karkkainen (1992), r_{m,i} for polyphase sequences may be well approximated as:

$$r_{m,i} \approx 2L^2 \sum_{l=1}^{L-1} |C_{m,i}(l)|^2 \tag{19}$$

Further, a more suitable metric for evaluating performance is the averaged BER over all users (Shi & Latva-aho, 2003). The BER(i) for ith desired user is given by:

$$BER(i) = \frac{1}{2} \operatorname{erfc} \left(\sqrt{SNR(i)} \right) = \frac{1}{2} \operatorname{erfc} \left(\frac{1}{\sqrt{\frac{N_0}{2E_b} + \frac{1}{6L^3} \left(\sum_{m=1}^M r_{m,i} \right)}} \right) \tag{20}$$

To illustrate the effectiveness of the proposed sequence sets (ZCZ (64, 4, 12)) in an asynchronous DS-CDMA system, Figure 3 depicts the BER performance as a function of SNR. This figure showcases the benefit of implementing the proposed sequence set within an asynchronous DS-CDMA system. From the figure mentioned above, it can be observed that an increase in SNR leads to a decrease in the BER. This demonstrates the direct relationship between signal quality and the accuracy of data transmission.

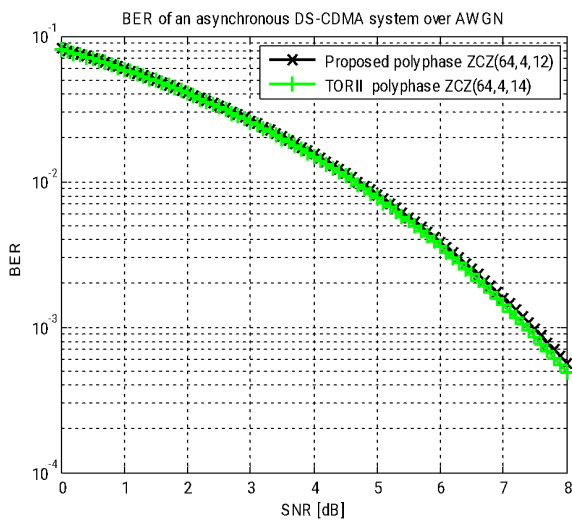


Figure 3. BER with respect to SNR of an asynchronous DS-CDMA system over AWGN. The performance of the proposed polyphase ZCZ sequences denoted as ZCZ (64, 4, 12), is compared with that of the TORII polyphase ZCZ sequences, denoted as ZCZ (64, 4, 14).

Further, the comparison analysis underscores the substantial benefits of adopting the proposed ZCZ construction featuring a Z_{CZ} zone of 12. Notably, despite the TORII ZCZ construction boasting a larger Z_{CZ} zone of 14, our approach achieves similar performance. The advantage stems from our proposed method's superior correlation properties. These sequences, derived from the MOCS matrix, demonstrably enhance system security due to their strong correlation characteristics. In contrast, the TORII method

relies on a basic DFT matrix, which may not provide the same level of security.

It is worth noting that making direct comparisons with other constructions is challenging due to notable differences in key parameters (L, M, and Z_{CZ}) utilized in the proposed approaches.

7. Conclusions

In this paper, a new class of quaternary Zero Correlation Zone (ZCZ) sequence sets based on MOCS matrix and polyphase perfect sequence has been proposed for quasi-synchronous CDMA systems to ensure interference-free communication. The obtained near optimal quadriphase ZCZ sequence has flexible parameters compared with the proposed approaches and can be constructed without using complex elements.

Conflict of interest

The authors have no conflict of interest to declare.

Funding

The authors received no specific funding for this work.

References

Abd, T. H., Aljunid, S. A., Fadhil, H. A., Junita, M. N., & Saad, N. M. (2012). Modelling and simulation of a 1.6 Tb/s optical system based on multi-diagonal code and optical code-division multiple-access. *Ukrainian Journal of Physical Optics*, 13 (2), 54-66.

Chen, D., Wang, J., Lu, H., Feng, L., & Jin, J. (2019). Experimental demonstration of quasi-synchronous CDMA-VLC systems employing a new OZCZ code construction. *Optics express*, 27(9), 12945-12956. <https://doi.org/10.1364/OE.27.012945>

Chen, X., Gao, X., & Peng, X. (2021). Construction of multiple optimal polyphase zero correlation zone sequence sets with inter-set zero cross-correlation zone. *IEEE Communications Letters*, 25(9), 2795-2799. <https://doi.org/10.1109/LCOMM.2021.3085312>

Chen, D., Wang, Q., Wang, J., Jin, J., Lu, H., & Feng, L. (2022). Performance evaluation of ZCC and OZCZ code set in an integrated VLCP-CDMA system. *IEEE Photonics Technology Letters*, 34(16), 846-849. <https://doi.org/10.1109/LPT.2022.3164702>

- Donelan, H., & O'Farrell, T. (2002). Large Families of Ternary Sequences with Aperiodic Zero Correlation Zone Sequences for a Multi-Carrier DS-CDMA System. In: *Electronics Letters*, 38(25) pp. 1666–1661.
- Driz, S., & Djebbari, A. (2019). Performance evaluation of sub-carrier multiplexed SAC-OCDMA system using optimal modulation index. *Journal of Optical Communications*, 40(1), 83-92.
<https://doi.org/10.1515/joc-2017-0044>
- Driz, S., Fassi, B., Kandouci, C., & Ghali, F. (2022). Performance improvement of a novel zero cross-correlation code using Pascal's triangle matrix for SAC-OCDMA systems. *Opto-Electronics Review*, e140550-e140550.
<https://doi.org/10.24425/opelre.2022.140550>
- Driz, S., & Fassi, B. (2022). Enhancing QS-SAC-OCDMA networks capacity via 2D spectral/polarization OZCZ coding technique based on modified Pascal's triangle matrix. *Optical and Quantum Electronics*, 54(8), 473.
<https://doi.org/10.1007/s11082-022-03865-8>
- Driz, S., Fassi, B., Mansour, M. A., & Taleb-Ahmed, A. (2023). FPGA implementation of a novel construction of optical zero-correlation zone codes for OCDMA systems. *Journal of Optical Communications*, 44(2), 195-204.
<https://doi.org/10.1515/joc-2019-0048>
- Fan, P. Z., Suehiro, N., Kuroyanagi, N., & Deng, X. M. (1999). Class of binary sequences with zero correlation zone. *Electronics Letters*, 35(10), 777-779.
<https://doi.org/10.1049/el:19990567>
- Fassi, B., Djebbari, A., & Taleb-Ahmed, A. (2014). Ternary zero correlation zone sequence sets for asynchronous DS-CDMA. *Communications and Network*, 6(4), 209-2017.
<https://doi.org/10.4236/cn.2014.64023>
- Fassi, B., & Taleb-Ahmed, A. (2018). A new construction of optical zero-correlation zone codes. *Journal of Optical Communications*, 39(3), 359-368.
<https://doi.org/10.1515/joc-2017-0214>
- Fassi, B., & Driz, S. (2023). New OZCZ Codes Based ZCC Constructions for VLC-QS-OCDMA Networks. *Wireless Personal Communications*, 132(2), 785-797.
<https://doi.org/10.1007/s11277-023-10621-y>
- Hayashi, T. (2007). Zero-correlation zone sequence set constructed from a perfect sequence. *IEICE transactions on fundamentals of electronics, communications and computer sciences*, 90(5), 1107-1111.
<https://doi.org/10.1093/ietfec/e90-a.5.1107>
- Hayashi, T., & Matsufuji, S. (2009). A generalized construction of optimal zero-correlation zone sequence set from a perfect sequence. In *2009 Fourth International Workshop on Signal Design and its Applications in Communications* (pp. 24-27). IEEE.
<https://doi.org/10.1109/IWSDA.2009.5346427>
- Hayashi, T., Maeda, T., Pham, A. T., & Matsufuji, S. (2018). A Novel Class of Structured Zero-Correlation Zone Sequence Sets. *IEICE Transactions on Fundamentals of Electronics, Communications and Computer Sciences*, 101(12), 2171-2183.
- Karkkainen, K. H. A. (1992). Mean-square cross-correlation as a performance measure for spreading code families. In *Proc. 1992 IEEE ISSSTA* (pp. 147-150).
<https://doi.org/10.1109/ISSSTA.1992.665668>
- Kumar, N., Majhi, S., & Jha, S. K. (2023). A new construction of almost-optimal multiple ZCZ sequence sets for multi-cell QS-CDMA system. *Cryptography and Communications*, 1-16.
<https://doi.org/10.1007/s12095-023-00684-w>
- Li, Y., Tian, L., & Liu, T. (2019). Constructions of polyphase ZCZ sequence sets with low cross-correlation property. *IET Communications*, 13(6), 733-740.
<https://doi.org/10.1049/iet-com.2018.5585>
- Li, X., Ding, Q. A., Nie, B., Liu, C., Wang, X., Song, J., ... & Zhang, L. (2020). Development and performance improvement of a novel zero cross-correlation code for SAC-OCDMA systems. *Journal of Optical Communications*, 44(s1), s1327-s1339.
<https://doi.org/10.1515/joc-2020-0086>
- Liu, W., Chen, X., Huang, N., & Xu, Z. (2024). Experimental Verification of a Multiuser Detection Technique for Asynchronous UOWC Systems. *IEEE Photonics Journal*.
<https://doi.org/10.1109/JPHOT.2024.3365695>
- Matsufuji, S., Hayashi, T., Kuroda, S., Ida, Y., & Matsumoto, T. (2015). On bent ZCZ codes. In *2015 Seventh International Workshop on Signal Design and its Applications in Communications (IWSDA)* (pp. 196-199). IEEE.
<https://doi.org/10.1109/IWSDA.2015.7458403>

- Mrabet, H., Cherifi, A., Raddo, T., Dayoub, I., & Haxha, S. (2020). A comparative study of asynchronous and synchronous OCDMA systems. *IEEE Systems Journal*, 15(3), 3642-3653.
<https://doi.org/10.1109/JSYST.2020.2991678>
- Ohira, Y., Matsumoto, T., Torii, H., & Ida, Y. (2022). Construction of Optical ZCZ Sequence Set with the Zero-Correlation Zone 2z Suitable for PPM-OCDMA System. In *2022 24th International Conference on Advanced Communication Technology (ICACT)* (pp. 106-111). IEEE.
<https://doi.org/10.23919/ICACT53585.2022.9728856>
- Ouis, E., Driz, S., & Fassi, B. (2022). Enhancing confidentiality protection for ZCZ-OCDMA network using line selection and wavelength conversion based on SOA. *Journal of Optical Communications*, 43(4), 563-572.
<https://doi.org/10.1515/joc-2020-0089>
- Pursley, M. (1977). Performance evaluation for phase-coded spread-spectrum multiple-access communication-Part I: System analysis. *IEEE Transactions on communications*, 25(8), 795-799.
<https://doi.org/10.1109/TCOM.1977.1093915>
- Sarangal, H., Nisar, K. S., Thapar, S. S., Singh, A., & Malhotra, J. (2021). Performance evaluation of 120 GB/s hybrid FSO-SACOCDMA-MDM system using newly designed ITM-Zero cross-correlation code. *Optical and Quantum Electronics*, 53, 1-12.
<https://doi.org/10.1007/s11082-020-02648-3>
- Shi, Q., & Latva-aho, M. (2003). Spreading sequences for asynchronous MC-CDMA revisited: Accurate bit error rate analysis. *IEEE transactions on communications*, 51(1), 8-11.
<https://doi.org/10.1109/TCOMM.2002.807631>
- Torii, H., Matsumoto, T., & Nakamura, M. (2013). Extension of methods for constructing polyphase asymmetric ZCZ sequence sets. *IEICE Transactions on Fundamentals of Electronics, Communications and Computer Sciences*, 96(11), 2244-2252.
<https://doi.org/10.1587/transfun.E96.A.2244>
- Torii, H. (2001). New method for constructing polyphase ZCZ sequence sets. In *Proc. 2nd WSEAS multiconference on applied and theoretical mathematics, 2001* (pp. 100-104).
<https://cir.nii.ac.jp/crid/1572261551258381440>
- Tseng, C. C., & Liu, C. (1972). Complementary sets of sequences. *IEEE Transactions on Information theory*, 18(5), 644-652.
<https://doi.org/10.1109/TIT.1972.1054860>
- Wang, Z., Ye, Z., Tang, C., & Yang, Y. (2024). New construction of optimal ZCZ sequence sets with inter-set zero cross-correlation zone. *Cryptography and Communications*, 16(1), 209-228.
<https://doi.org/10.1007/s12095-023-00685-9>
- Wysocki, B. J. (2003). On a method to improve correlation properties of orthogonal polyphase spreading sequences. *Journal of Telecommunications and Information Technology*, (2), 99-105.
<https://jtit.pl/jtit/article/view/164>