A novel quadriphase ZCZ sequences for QS-CDMA systems

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Abstract: In this article, a novel method for constructing quadriphase Zero Correlation Zone (ZCZ) sequence sets based on the mutually orthogonal complementary sets (MOCS) matrix and polyphase perfect sequence is presented. Based on the obtained MATLAB results in terms of correlation functions, it can be mentioned that the resultant ZCZ set is suitable for quasi-synchronous Code Division Multiple Access (QS-CDMA) systems. The obtained sequence sets are near-optimal concerning mathematical bound, and their construction is more flexible than other polyphase ZCZ constructions. Furthermore, their parameters can be chosen flexibly.

Keywords: MOCS matrix, perfect sequence, QS-CDMA, ZCZ codes
1. Introduction

Code Division Multiple Access (CDMA) is a promising channel access method allowing a multiuser communication scheme where multiplexed users simultaneously share the same access network. In such systems, unique signature sequences are dedicated to different users to ensure secure network connection, flexibility, and multimedia services. Several CDMA categories have been proposed such as synchronous (Abd et al., 2012; Driz & Djebarri, 2019; Driz et al., 2023; Mrabet et al., 2020), asynchronous (Liu et al., 2024), and quasi-synchronous (QS) architectures (Fassi & Driz, 2023; Kumar et al., 2023) applied to different network scenarios. Accordingly, different coding schemes with adapted correlation properties have been developed. In a synchronous OCDMA system where no relative time delay between the user’s signals is considered ($\tau = 0$), spreading sequences with zero cross correlation (ZCC) properties are required to avoid multiple access interference (MAI) (Driz et al., 2022; Li et al., 2020; Sarangal et al., 2021).

Nevertheless, in some situations where $\tau \neq 0$ (asynchronous systems), code sequences with Zero Correlation Zone (ZCZ) properties are needed to eliminate the MAI effects (Fassi & Taleb-Ahmed, 2018; Wang et al., 2024).

Another CDMA system (quasi-synchronous architecture) is defined when $\tau \neq 0$ in some limited time range, smaller than the sequence length. In such architecture, the ZCC codes are the suitable choice where the maximum delay is well known as the ZCZ zone (Chen et al., 2019; Driz & Fassi, 2022). On the other hand, several ZCZ code constructions such as binary, ternary, polyphase, and optical codes have been proposed. Binary ZCZ codes, characterized by binary symbols typically denoted as -1 and 1, face challenges in simultaneously increasing:

- Sequence length $L$ improves noise immunity but reduces data rate and potentially increases complexity,
- Set size $M$ allows for more users but might require reduced ZCZ length,
- Zero-correlation length ZCZ: reduces interference but comes at the cost of potentially longer sequences.

These parameters are all interconnected, and there is a fundamental trade-off to be made between these factors depending on the specific needs of the communication system. Thus, constructing optimal ZCZ sequences tailored for communication systems necessitates specific criteria - mathematical bounds- to help determine the theoretical limits for achieving good ZCZ properties with a given sequence length effects (Fassi & Taleb-Ahmed, 2018). In contrast, ternary ZCZ codes, utilizing three levels usually labeled as -1, 0, and 1, were developed to address this challenge by enhancing the set size to reach a more relaxed upper bound in terms of the sequence set compared to their binary counterparts. The theoretical upper bound, contingent upon the phase shift’s absolute value being less than or equal to ZCZ width, is expressed as $M.(Zcz + 1) \leq L$, called the Tang-Fan-Matsufuji bound (Donelan & O’Farrell, 2002; Fassi et al., 2014; Hayashi et al., 2018; Matsufuji, et al., 2015).

Meanwhile, polyphase ZCZ codes incorporate complex symbols, expanding beyond simple binary or ternary representations. Polyphase ZCZ codes introduce a rich tapestry of symbols, including not only 1 and -1 but also 0, j, -j, and potentially an array of other complex elements. This expansion into the domain of complex symbols endows polyphase ZCZ codes with unprecedented versatility and adaptability. One of the standout features of polyphase ZCZ codes is their inherent capacity to accommodate expanding sets of codes with varying sizes. This means that instead of being confined to fixed-length sequences, polyphase ZCZ codes can seamlessly adapt to generate code sets of diverse lengths. This flexibility is instrumental in addressing the multifaceted demands of modern communication systems, where requirements can vary widely depending on factors such as channel conditions, bandwidth constraints, and signal processing capabilities (Chen, et al., 2021; Li et al., 2019; Torii et al., 2013). Moreover, Polyphase ZCZ codes have the property that their maximum autocorrelation values are typically equal to the length of the sequence at a specific shift – shift 0. This characteristic is desirable in code design as it helps minimize interference between different sequences. Furthermore, the incorporation of polyphase sequences enhances the security level of the system, as their complex construction methods make interception difficult. This reinforces their suitability for robust communication applications.

On the other hand, optical ZCZ (OZCZ) has been adapted and applied to optical communication to encode data onto optical signals, typically in the form of pulses or modulated light. This adaptation serves to mitigate interference and bolster signal integrity, rendering OZCZ a pivotal strategy for optimizing the efficacy of quasi-synchronous optical communication systems (Chen et al., 2022; Driz et al., 2023; Ohira et al., 2022; Ouis et al., 2022).

In this paper, we propose a novel method for constructing quadriphase sets of ZCZ complex spreading sequences based on the MOCS matrix and polyphase perfect sequence to achieve correlation properties suited for QS-CDMA applications. The remainder of this paper is organized as follows. Section 2 gives some definitions and nomenclature required for the rest of the document. Sets of sequences with desired correlation properties are constructed by using the proposed method in Section 3. Section 4 provides an example of a quadriphase ZCZ spreading sequence followed by the properties of the proposed code in Section 5. An examination of the system's performance concerning Bit Error Rate (BER) is detailed in Section 6. Conclusions concludes the study.
2. Preliminary considerations

2.1. Periodic correlation function

The periodic correlation function (PCF), $\theta(b_i, b_i') (\tau)$ between a pair of binary sequences of length $L$ at a lag $\tau$ is defined by Driz et al. (2023):

$$\forall \tau \geq 0, \quad \theta(b_i, b_i') (\tau) = \sum_{j=0}^{L-1} b_{ij} b_{ij'} \mod(L)$$

(1)

2.2. Aperiodic correlation function

The aperiodic correlation function (ACF), $\varphi(b_i, b_i') (\tau)$ for general polyphase sequences of length $L$ is defined as follows by Li et al., 2019; Wysocki, 2003:

$$\varphi(b_i, b_i') (\tau) = \begin{cases} 
\frac{1}{L} \sum_{j=0}^{L-1} b_{ij} b_{ij'} (j, \tau) & 0 \leq \tau \leq L - 1 \\
\frac{1}{L} \sum_{j=0}^{L-1} b_{ij} b_{ij'} (j, \tau) & 1 - L \leq \tau < 0 \\
0 & |\tau| \geq L 
\end{cases}$$

(2)

where $[\cdot]^*$ denotes a complex conjugate operation.

2.3. ZCZ sequences

A set of $M$ sequences is called an optical Zero Correlation Zone (ZCZ) sequence set, denoted by $ZCZ(L, M, ZCZ)$, where $ZCZ = \min(Z_a, Z_c)$ indicate the Zero Correlation Zone if the periodic correlation function satisfies (Driz et al., 2023):

$$\theta(b_i, b_i') (\tau) = \begin{cases} 
\omega & \tau = 0, i = i' \\
0 & i = i', 1 \leq |\tau| \leq Z_a \\
0 & i \neq i', |\tau| \leq Z_c 
\end{cases}$$

(3)

2.4. Perfect sequences

A sequence $b = (b_0, b_1, ..., b_{L-1})$ of length $L$ is said to be a perfect sequence if the ideal periodic autocorrelation function (PACF) is given by (Hayashi, 2007; Hayashi & Matsufuji, 2009):

$$\theta_{bb}(\tau) = \sum_{i=0}^{L-1} b_{i} b_{i+\tau} = \begin{cases} 
\eta L_1, \eta \leq L & \text{for } \tau = 0 \\
0 & \text{for } 1 \leq \tau \leq L_1 - 1 
\end{cases}$$

(4)

where $\eta$ denotes energy efficiency.

3. Quadriphase ZCZ code construction

The construction procedure of the proposed quadriphase OZCZ codes is performed in three steps.

3.1. Step 1

First, let $F^{(n)} (n \geq 0)$ denote a MOCS matrix of order $(M \times M) = (2^{n+1} \times 2^{n+1})$ where each element of $F^{(n)}$ is a sequence of length $l_e = 2^n \times l_m = 2^{m+n}$ $(l_m = 2^m, m \geq 0)$ and each line have a length of $L = 2^{2n+m+1}$. Thus, an MOCS matrix of $(2^{2n+1} \times 2^{2n+m+1})$ where $n \geq 0$ and $m \geq 0$ is arranged as follows (Fan et al., 1999):

$$F^{(n)} = \begin{bmatrix}
F_{11} & F_{12} & \cdots & F_{1k} & F_{1M} \\
F_{21} & F_{22} & \cdots & F_{2k} & F_{2M} \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
F_{M1} & F_{M2} & \cdots & F_{Mk} & F_{MM}
\end{bmatrix}_{M \times M} = \begin{bmatrix}
f_0 & f_1 & \cdots & f_k & f_M \\
f_0 & f_1 & \cdots & f_k & f_M \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
f_0 & f_1 & \cdots & f_k & f_M \\
\end{bmatrix}_{M \times M}$$

(5)

Further, $F^{(n)}$ can be considered as a MOCS matrix when it satisfies the following formulas (Tseng & Liu, 1972):

$$\sum_{k=1}^{M} \varphi(F_{ik}, F_{ik}) (\tau) = 0, \quad \forall i, \forall \tau \neq 0$$

$$\sum_{k=1}^{M} \varphi(F_{ik}, F_{ik'} (\tau) = 0, \quad \forall i \neq i', \forall \tau \neq 0$$

(6)

(7)

Where $\varphi(F_{ik}, F_{ik}) (\tau)$ and $\varphi(F_{ik}, F_{ik'} (\tau)$ are the aperiodic autocorrelation and aperiodic cross-correlation functions, respectively.

3.2. Step 2

Let $A_l = (a_0, a_1, ..., a_l-1)$ denote a perfect sequence of length $l$ ($l$ must be an even integer and $0 \leq i \leq l$) whose elements are complex numbers having absolute value 1. Let $l_0$ and $l_1$ be two integers such that $l = l_1 \times l_0$ $(1 \leq l_1 < l$ and $1 \leq l_0 < l$).

By using the elementary transformation for perfect sequences that shifts the sequence cyclically to the left by $(i \times l_0)$, we can generate more than one perfect sequence. The set of $l_1$ generated perfect sequences is denoted by $A$:

$$A = \begin{bmatrix}
A_0 \\
A_1 \\
\vdots \\
A_{l_1-1}
\end{bmatrix} = \begin{bmatrix}
a_0 & a_1 & \cdots & a_{l_1-1} \\
a_0 & a_1 & \cdots & a_{l_1-1} \\
\vdots & \vdots & \ddots & \vdots \\
a_0 & a_1 & \cdots & a_{l_1-1}
\end{bmatrix} = [b_0 \ b_1 \ b_2 \ ... \ b_{l-1}]$$

(8)

where $b_s = \begin{bmatrix} a_0 \\
b_1 \\
\vdots \\
a_s \\
\end{bmatrix}$,
next, we convert the matrix $A$ into a single-column matrix denoted by $B$ as follows:

$$B = \begin{bmatrix} b_0 \\ b_1 \\ \vdots \\ b_s \\ b_{l-1} \end{bmatrix}$$  \hspace{1cm} (9)

The order of elements in the matrix $B$ is by column. Indeed, the values are taken column by column from the matrix $A$ by moving down one column and then across to the right to the next column. The obtained $B$ matrix contains $(l \ast l_1) - 1$ rows.

3.3. Step 3

Using the elements of the matrix $B$, we generate the matrix $Q_r$ of length $(l \ast l_1 \ast l_e)$ by concatenating each element in the matrix $B$ $l_e$ times, as follows:

$$Q_r = [B(r) \cdots B(r)]_{l_e \text{ times}}$$  \hspace{1cm} (10)

where $B(r)$ denotes the $r^{th}$ element of the matrix $B$. Next, we convert matrix $Q_r$ to one dimensional array matrix denoted $Q$ of length $(l_e \ast l_1 \ast l)$ by concatenating horizontally all obtained rows of the matrix $Q_r$ as follows:

$$Q = [Q_0 Q_1 Q_2 \cdots \cdots Q_{(l_1+1)-2} Q_{(l_1+1)-1}]$$  \hspace{1cm} (11)

On the other hand, let us define $D_i$ sequence obtained from the MOCS matrix $F^{(n)}$ by concatenating its $i^{th}$ arrays $(f_i)$ horizontally $l$ times as follows:

$$D_i = [(f_i f_i \cdots f_i)]_{l \text{ times}}$$  \hspace{1cm} (12)

Finally, the obtained quaternary set $C$ contains $C_i$ sequences, each of length $(l \ast L)$, given by:

$$C = \begin{bmatrix} C_0 \\ \vdots \\ C_i \\ \vdots \\ C_{l-1} \end{bmatrix} = \begin{bmatrix} c_0^0 & \ldots & c_0^j & \ldots & c_0^{(L-1)} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ c_i^0 & \ldots & c_i^j & \ldots & c_i^{(L-1)} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{l-1}^0 & \ldots & c_{l-1}^j & \ldots & c_{l-1}^{(L-1)} \end{bmatrix}$$  \hspace{1cm} (13)

where $0 \leq i \leq l_1 - 1$ and $0 \leq j \leq (l \ast L) - 1$.

The quaphase sequences can be derived from Equations 11 and 12 as follows:

$$C_i = D_i \otimes Q$$  \hspace{1cm} (14)

$$c_i^j = f_i,j \mod l, Q_j$$  \hspace{1cm} (15)

where $\otimes$ denotes the element-by-element multiplication and $c_i^j$ is the $j^{th}$ element of $C_i$.

4. Example of quaphase ZCZ spreading sequences

4.1. Step 1

We choose $F^{(1)}$ a MOCS matrix of $(M \times M) = (4 \times 4)$ (with $n = 1$ and $m = 0$), obtained by using Equation 5 as follows:

$$F^{(1)} = \begin{bmatrix} + & - & - & - & + & - & + & - \\ + & + & + & + & - & + & + & - \\ - & + & - & + & + & + & - & - \\ + & + & + & + & + & + & + & - \\ - & - & + & - & - & - & + & - \end{bmatrix} = \begin{bmatrix} f_0 \\ f_1 \\ f_2 \\ f_3 \end{bmatrix}$$

where $(-)$ and $(+)$ denote $(-1)$ and $(+1)$, respectively. In this case, $l_e = 2^{m+n} = 2$ and $L = 2^{2m+n+1} = 8$.

4.2. Step 2

In this step, we choose a perfect sequence, denoted by $A_0$, of length $l = 8$ ($l_1 = 4$ and $l_0 = 2$) given by: $A_0 = (1,1,j,1,1,-1,j,-1)$. For $0 \leq i \leq 3$, we can construct four (04) perfect sequences of length $l = 8$ as follows:

$$A_0 = (00100212), A_1 = (10021200), A_2 = (02120010), A_3 = (12001002).$$

Where $0, 1, and 2$ denote $1, j, -1$ respectively and $j = \sqrt{-1}$.

From the obtained sequences, we apply Equation 8 to find the matrix $A (4 \times 8)$ as follows:

$$A = \begin{bmatrix} A_0 \\ A_1 \\ A_2 \\ A_3 \end{bmatrix} = \begin{bmatrix} 00100212 \\ 10021200 \\ 02120010 \\ 12001002 \end{bmatrix} = [b_0 b_1 b_2 b_3 b_4 b_5 b_6 b_7]$$
Where:
\[
b_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} ; b_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} ; b_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} ; \ldots \ldots ; b_6 = \begin{bmatrix} 1 \\ \vdots \\ 0 \\ 0 \\ 0 \\ 2 \end{bmatrix} ; b_7 = \begin{bmatrix} 2 \\ 2 \\ \vdots \end{bmatrix} \]

Similarly, the matrix \( B (32 \times 1) \) is obtained from Equation 9 as follows:
\[
B = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 2 \\ 2 \\ \vdots \\ 1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 2 \\ 2 \end{bmatrix}
\]

4.3. Step 3
For \(0 \leq r \leq 31\), we use Equations 10 and 11 to find the matrix \( Q_r (32 \times 2) \) and \( Q (64 \times 1) \) as given below:
\[
Q_r = \begin{bmatrix} 0 & 0 \\ 1 & 1 \\ 0 & 0 \\ 1 & 1 \\ 0 & 2 \\ 2 & 2 \\ \vdots & \vdots \\ 0 & 0 \\ 2 & 2 \end{bmatrix},
\]
where:
\[
Q_0 = [00]; Q_1 = [11], Q_2 = [00], \ldots, Q_{30} = [00], Q_{31} = [22] \text{ and } Q = [00110011000022221100110000222200001101112220000110011002222000022],
\]

For \(0 \leq i \leq 3\) and using Equation 12, the sequences \( D_i \) are given by:
\[
C_0 = (00322013002202201122310200002120033201322002021122310222222021) \]
\[
C_1 = (00112031000002021110031200022022000112031222220110212022020022),
\]
\[
C_2 = (2013003320220203102112220222222010303330220002231021122020200000),
\]
\[
C_3 = (203100112022022231201100222202020310011020200031211002222000222),
\]

where \(0, 1, 2, 3\) denote \(1, j, -1, -j\) respectively and \(j = \sqrt{-1}\).

The correlation functions (PACF and PCCF periodic cross-correlation function) of the obtained sequences are illustrated in Figure 1 and Figure 2.
5. Properties of the proposed code

The proposed quadriphase ZCZ code can be generated from the MOCS matrix and polyphase perfect sequence. The generated sequence set satisfies the following properties:

\[
\begin{align*}
\forall i, \forall \tau \neq 0, |\tau| &\leq (l-2)2^{n+m} \\
\theta(C_i, C_\tau)(\tau) &= 0
\end{align*}
\]

and

\[
\begin{align*}
\forall i \neq \nu, \forall |\tau| &\leq (l-2)2^{n+m} \\
\theta(C_i, C_\nu)(\tau) &= 0
\end{align*}
\]

\( \{C\} \) is the set of ZCZ sequences with parameters \((N, M, Z_{CZ}) = ((l2^{n+m+1}), 2^n + 1, (l-2)2^{n+m}) \). The parameters of the proposed construction satisfy the theory of the bound of ZCZ codes. Let be the ratio of optimality:

\[
\rho = \frac{M(C_{ZCZ} + 1)}{N} = \frac{2^{n+1}([l-2]2^{n+m} + 1)}{l2^{2n+m+1}} = 1 - \frac{2}{l} + \frac{1}{l2^{2n+m}} < 1.
\]

Table 1 gives the bound of the proposed construction according to the parameters \((n, m)\) of the MOCS matrix and the length \(l\) of perfect sequence. From Table 1, we note that when the length of the perfect sequence increases, the optimal ratio of proposed sequences approaches the theoretical limit of ZCZ sequences.

<table>
<thead>
<tr>
<th>(l)</th>
<th>2</th>
<th>8</th>
<th>16</th>
<th>32</th>
</tr>
</thead>
<tbody>
<tr>
<td>(n + m = 0)</td>
<td>0.50</td>
<td>0.87</td>
<td>0.93</td>
<td>0.96</td>
</tr>
<tr>
<td>(n + m \neq 0)</td>
<td>0.25</td>
<td>0.81</td>
<td>0.90</td>
<td>0.95</td>
</tr>
<tr>
<td>((n = 1, m = 0))</td>
<td>0.12</td>
<td>0.78</td>
<td>0.89</td>
<td>0.94</td>
</tr>
<tr>
<td>((n = 0, m = 1))</td>
<td>0.12</td>
<td>0.78</td>
<td>0.89</td>
<td>0.94</td>
</tr>
</tbody>
</table>

6. System analysis

In DS CDMA systems, minimizing the maximum values of aperiodic cross-correlation functions and out-of-phase aperiodic autocorrelation functions is required. Pursley (1977) demonstrated that for a BPSK asynchronous DS CDMA system, the average signal-to-noise ratio (SNR) at the receiver output of the \(i^{th}\) user’s correlator receiver can be formulated in terms of the average interference parameter (AIP) from the other \(M\) users in the system and the power of white Gaussian noise within the channel.

The SNR for \(i^{th}\) desired user denoted as \(SNR(i)\), can be expressed in the form:

\[
SNR(i) = \left[\frac{N_0}{2E_b} + \frac{1}{L^2} \left(\sum_{m=1}^{M} r_{m,i}\right)\right]^{-0.5}
\]

where \(E_b\) is the bit energy, \(N_0\) is one-sided Gaussian noise power spectral density, and \(r_{m,i}\) is the average interference parameter (AIP), defined for a pair of sequences as:

\[
r_{m,i} = 2\mu_m(0) + Re\{\mu_m(1)\}.
\]

The cross-correlation parameters \(\mu_{m,i}(\tau)\) are defined as:

\[
r_{m,i} = L^2 \sum_{l=1}^{L} C_m(l) [C_m(l + \tau)]^* \]

where \(C_m(l) = \phi(b_m, b_l)(l)\) represents the aperiodic correlation function (ACF), for general polyphase sequences of length \(L\), defined in Equation 2.

However, following the derivation in Karkkainen (1992), \(r_{m,i}\) for polyphase sequences may be well approximated as:
Further, a more suitable metric for evaluating performance is the averaged BER over all users (Shi & Latva-aho, 2003). The BER(i) for ith desired user is given by:

\[
\text{BER}(i) = \frac{1}{2} \text{erfc} \left( \sqrt{\text{SNR}(i)} \right)
\]

(20)

To illustrate the effectiveness of the proposed sequence sets (ZCZ (64, 4, 12)) in an asynchronous DS-CDMA system, Figure 3 depicts the BER performance as a function of SNR. This figure showcases the benefit of implementing the proposed sequence set within an asynchronous DS-CDMA system. From the figure mentioned above, it can be observed that an increase in SNR leads to a decrease in the BER. This demonstrates the direct relationship between signal quality and the accuracy of data transmission.

Further, the comparison analysis underscores the substantial benefits of adopting the proposed ZCZ construction featuring a \( Z_{\text{CZ}} \) zone of 12. Notably, despite the TORII ZCZ construction boasting a larger \( Z_{\text{CZ}} \) zone of 14, our approach achieves similar performance. The advantage stems from our proposed method’s superior correlation properties. These sequences, derived from the MOCS matrix, demonstrably enhance system security due to their strong correlation characteristics. In contrast, the TORII method relies on a basic DFT matrix, which may not provide the same level of security.

It is worth noting that making direct comparisons with other constructions is challenging due to notable differences in key parameters (L, M, and \( Z_{\text{CZ}} \)) utilized in the proposed approaches.

7. Conclusions

In this paper, a new class of quaternary Zero Correlation Zone (ZCZ) sequence sets based on MOCS matrix and polyphase perfect sequence has been proposed for quasi-synchronous CDMA systems to ensure interference-free communication. The obtained near optimal quadrature ZCZ sequence has flexible parameters compared with the proposed approaches and can be constructed without using complex elements.

Conflict of interest

The authors have no conflict of interest to declare.

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References


r_{m,i} \approx 2L^2 \sum_{l=1-L}^{L-1} |c_{m,i}(l)|^2. \quad (19)


