Journal of Applied Research and Technology

www.jart.icat.unam.mx

Journal of Applied Research and Technology 19 (2021) 364-378

Original

A multi-item manufacturing system with an outsourcer and random defective items

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Received 07 18 2020; accepted 04 20 2021
Available 08 31 2021

Abstract: This study examines a multi-item manufacturing problem with a single machine, an outsourcer, and random defective items. To cope with the increasing multi-product demands from global markets, modern manufacturing firms must make an efficient production plan to satisfy customer's needs with quality goods and smooth the in-house fabrication schedule and utilization. Outsourcing is an effective option to avoid machine overloads and smooth fabrication schedules. Further, the fabrication of random defective products is inevitable because of unforeseen factors in real manufacturing environments. These products must be identified, separated, and discarded to retain the finished lot's desired quality. To address the above-mentioned concerns, this study develops a mathematical model to represent a hybrid stock refilling system, employs mathematical derivations to find long-run average system expenses, and uses an optimization technique to derive a closed-form common rotation time for this hybrid system. The results of this study show the individual and combined impacts of variations in outsourcing percentages and scrap rates on optimal rotation time and diverse core system parameters (such as machine utilization, specific cost component, etc.) to facilitate planning, controlling, and decision making in such a particular hybrid fabrication system.

Keywords: Multi-item manufacturing, outsourcer, hybrid refilling system, rotation cycle time, imperfect fabrication, scrap

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Peer Review under the responsibility of Universidad Nacional Autónoma de México.

1. Introduction

This study examines a multi-item manufacturing problem with a single machine, an outsourcer, and random defective items. To help production managers decide the most economical lot size, Taft (1918) proposed the economic production quantity model for a single-item fabrication under a perfect fabrication condition. Using mathematical modeling and balancing the setup and stock holding costs, author determined the most economic batch size to minimize long-run average fabrication-related costs. Facing the trend of increasing multiproduct demands from global markets, modern manufacturing firms have focused on making an efficient multi-item fabrication plan to meet customers' needs. Aggarwal (1984) suggested a simplified common order cycle for subgroups of multi-item inventories and proposed an efficient computational procedure to find the optimal order cycle values. Leachman and Gascon (1988) examined a multiproduct single-machine manufacturing system with time-varying stochastic demands. Based on the results from observing stock status linking to the maintenance of economic production cycles, the authors proposed a dynamic cycle-length heuristic to consolidate the feedback controls to facilitate the production decision making on a period time basis. Bruggemann and Jahnke (1994) considered a two-stage multi-item scheduling and lot-sizing problem. The authors incorporated the partially non-linear constraints into a conventional mixed-integer linear model to deal with their two-stage batch production. Since it turned into NP-complete in positive setup times, the authors used the simulated annealing approach for calculating fabrication schedules on both stages. Khoury et al. (2001) used a common cycle method to explore a multiproduct lot-scheduling problem with insufficient capacity. The authors started examining a problem with two products and extended to problems with multiple products using a common cycle approach with discussions. Rossetti and Achlerkar (2011) assessed segmentation techniques for managing massive scale multi-item inventories. The authors developed two different plans that used statistical clustering, namely (i) grouped multi-item individual policies and (ii) multi-item group policies, and evaluated their performance via a set of experiments to conclude that they can outperform the inventory management result from using ABC analysis. Additional studies (Chiu, Wu & Tseng, 2019; Chiu, Huang, et al., 2019; Chiu, Lin & Wu, 2020; Gallego et al., 1996; Keshavarzfard et al., 2019; Lesmono et al., 2020 Pochet & Wolsey, 1991; Taleizadeh et al., 2019) examined multi-item fabrication systems with diverse features.

For production management to smooth fabrication schedules and avoid machine overloads, outsourcing is an effective option. de Kok (2000) examined allocation and outsourcing strategies for packaging capacity in a process industry, wherein the limited capacity is allocated among various package sizes; each has a targeted fill rate. According to a periodic review ordering policy, the author proposed two separate capacity reservation strategies: one is to postpone excess capacity needs into the future, and the other is to outsource the excessive capacity needs. Then, cost minimized optimal policies were examined, and the best policy was selected. The author also offered managerial insights into the impact of diverse cost variables and processes and strategy selection. Yildirim et al. (2005) examined a multi-period stochastic fabrication planning and sourcing problem under service level constraints. A producer has multiple plants and/or subcontractors, and each source has its own capacity, lead time, and fabrication cost. The firm must satisfy the random demands of multiproduct according to preset service level requirements from its clients. The authors employed mathematical programming to deal with the problem to determine the production quantity, when and where to produce them, and the number of inventories. Lee and Choi (2011) examined a two-stage fabrication scheduling problem incorporating an outsourcing plan, wherein two operations are required in each activity in these stages. Transactions can be completed either by an in-house facility or through an outside contractor. Their purpose was to minimize the weighted sum of the outsourcing expense and the in-house makespan. Due to the problem's NP-hard nature, the authors proposed an approximation algorithm to solve the problem. Hahn et al. (2016) explored multi-criteria outsourcing decision-making for stochastic manufacturing systems. For determining beneficial outsourcing alternatives at the strategic level, the authors presented a non-parametric ranking method using both non-model-based Key Performance Indicators (KPI) and model-based KPIs. To evaluate strategic outsourcing policy's performance, the authors offered an improved aggregate planning incorporating a queuing network to handle manufacturing systems' stochastic conduct. A real case form industry was utilized to demonstrate the potential benefits of their approach. Other studies (Chiu, Zhao et al., 2020; Dan-Asabe et al., 2019; Prajapati et al., 2020) explored the benefits, risks, and different aspects of implementing outsourcing policies in manufacturing firms.

Furthermore, the fabrication of random defective products is inevitable because of unanticipated factors in real manufacturing environments. These products must be identified, separated, and discarded to retain the desired quality of the finished lot. Wee (1993) examined inventory systems with permitted partial backorder for deteriorating products. Modeling and formulation were proposed along with two numerical examples for demonstrating how the author's policy can lead to minimum cost. Bertsimas and

Paschalidis (2001) studied the make-to-order production systems incorporating probabilistic service level requirements. They aimed to determine a fabrication plan to meet customer's multiproduct demand with minimum cost and under stochastic service level guarantees. Additional studies that addressed fabrication systems with different features of imperfect quality and unreliable issues can also be found (Adazabra et al., 2018; Afshar-Nadjafi et al., 2019; Delgoshaei & Ali, 2019; de Vasconcelos et al., 2019; Fathallah et al., 2019; Gabitov et al., 2020; Mandić et al., 2019; Mustajib et al., 2019; Ortiz-Servin et al., 2019; Sinha et al., 2020; Urrea & Pascal, 2018;). This study examines a multi-item manufacturing problem with a single machine, an outsourcer, and random defective items. Prior research paid little attention to the collective influences of outsourcing and scrap on multi-item systems. The present work aims to bridge the gap.

2. Problem description and assumption

The RHA steel plates of different thicknesses were produced at JINDAL Steel plant, Angul, Orissa. The process layout for the production of steel is shown in Fig. 1. Steel production started with melting directly reduced iron (DRI) in an electric arc furnace. Other raw materials include in plant scrap and the fluxes like lime and dolomite for making slag. The molten metal was transferred to a ladle furnace for final refining and was further subjected to vacuum degassing to minimise Hydrogen and other impurities. Table 1 illustrates the analysed final chemical composition of the steel.

A multi-item replenishing system with an outsourcer and random scraps is investigated. Consider L end products are made on a machine obeying a common cycle time discipline, at production rates P_{1i} (where i = 1, 2, …, L). Demands of these products are λ_i per year and to cut down the manufacturing cycle time, an outsourcer is used to help supply a π_i portion of the lot for each end item i in a cycle and these outsourced items are scheduled to be received at the time in-house production processes end (Figure 1). The relevant costs relating to paying outsourcer for each product i are the fixed cost $K_{\pi i}$ and unit cost C_π. The outsourcer is requested to guarantee product quality and timely delivery of all of its supplies. The outsourced items are to be received at the fabrication finishing point of each product i, which bring the inventory level of end product i from H_{1i} to H_i, before the beginning of depletion time $t_{2i\pi}$. Then, all finished items of product i are depleted before next fabrication cycle starts (Figure 1). Extra notations utilized in this study are given below:

Q_i = batch size of product i,
K_i = setup cost for manufacturing product i in-house,
C_i = unit in-house manufacturing cost,
β_{1i} = linking parameter between $K_{\pi i}$ and K_i, where $K_{\pi i} = (1 + \beta_{1i})K_i$ and $-1 < \beta_{1i} < 0$ is assumed,
β_{2i} = linking parameter between $C_{\pi i}$ and C_i, where $C_{\pi i} = (1 + \beta_{2i})C_i$ and $\beta_2 > 0$ is assumed,
C_{Si} = unit disposal cost of product i,
d_{1i} = production rate of defective product i,
x_i = random defective rate in the manufacturing process of product i,
$E[x_i]$ = the expected value x_i,
T_π = common manufacturing cycle time – the decision variable;
$E[T_\pi]$ = the expected value T_π,
H_{1i} = level of end product i in the end of in-house manufacturing process,
H_i = level of finished product i in the end of in-house manufacturing process and after the receipt of outside supplies,
h_i = unit holding cost of product i,
$I(t)_i$ = level of end products at time t,
$I_D(t)_i$ = level of defective items at time t,
$t_{1i\pi}$ = manufacturing uptime of product i,
$t_{2i\pi}$ = depletion time of product i,
t_{1i} = manufacturing uptime of product i for the same system without an outsourcer,
t_{2i} = depletion time of product i for the same system without an outsourcer,
T = the common manufacturing cycle time for the same system without an outsourcer,
$TC(T_\pi)$ = total system cost per cycle,
$E[TCU(T_\pi)]$ = the expected annual system cost,
$\bar{\pi}$ = the average of π_i
$\bar{x}$ = the average of x_i
$\overline{\beta_1}$ = the average of β_{1i},
$\overline{\beta_2}$ = the average of β_{2i}

As shortages are not allowed, $P_{1i} - d_{1i} - \lambda_i$ must be greater than zero. During the production processes of the other $(1 - \pi_i)$ portion of product i, an x_i proportion of defective items are produced at the rate d_{1i} randomly (hence, we have $d_{1i} = x_iP_{1i}$). The inventory level of defective product i produced in the proposed multi-item replenishing problem is illustrated in Figure 2.

3. Mathematical modeling and optimal solution

From the description of the proposed multiproduct replenishment system, formulas for batch size Q_i, common manufacturing cycle time T_π, and additional equations can be identified as follows (where i = 1, 2, …, L):

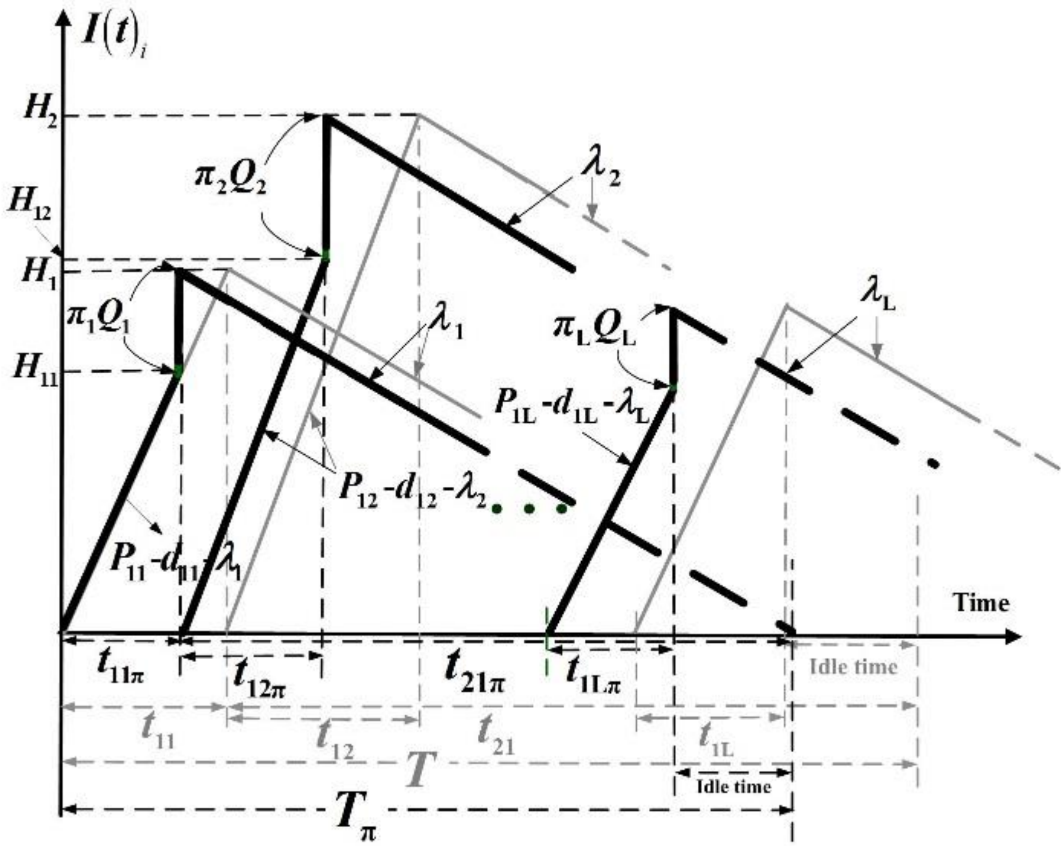

Figure 1. Inventory level of end product i in the proposed multi-item replenishing system considering an outsourcer and random defective items.

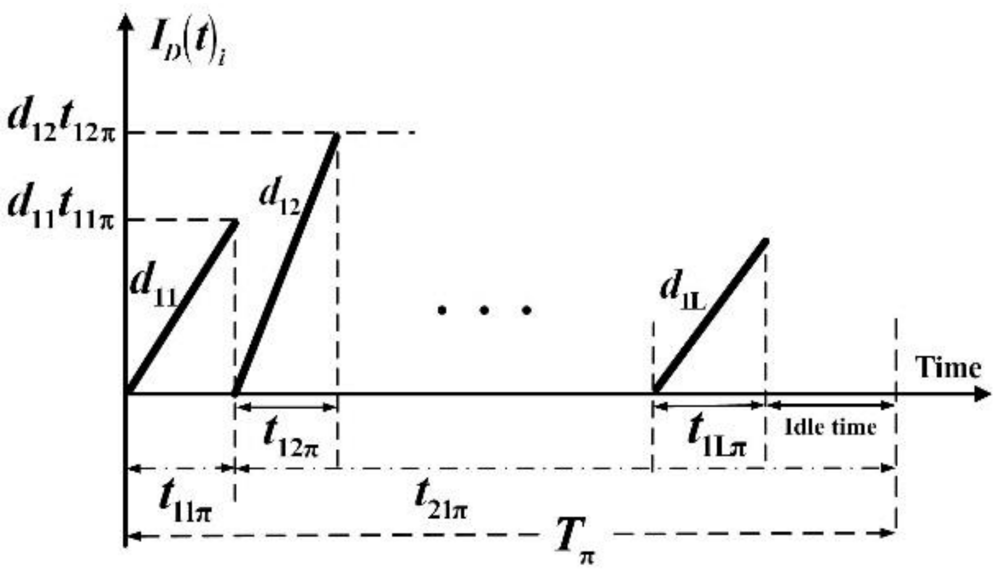

Figure 2. Inventory level of defective product i in the proposed multi-item replenishing problem.

$$Q_i = \frac{\lambda_i T_\pi}{\left[1 - x_i\left(1-\pi_i\right)\right]} \tag{1}$$

$$T_\pi = t_{1i\pi} + t_{2i\pi} \tag{2}$$

$$t_{1i\pi} = \frac{\left(1-\pi_i\right)Q_i}{P_{1i}} = \frac{H_{1i}}{P_{1i} - d_{1i} - \lambda_i} \tag{3}$$

$$t_{2i\pi} = \frac{H_i}{\lambda_i} \tag{4}$$

$$H_{1i} = \left(P_{1i} - d_{1i} - \lambda_i\right) t_{1i\pi} \tag{5}$$

$$H_i = H_{1i} + \pi_i Q_i = \lambda_i \cdot t_{2i\pi} \tag{6}$$

$$d_{1i} t_{1i\pi} = x_i P_{1i} t_{1i\pi} = x_i \left[\left(1-\pi_i\right)Q_i\right]. \tag{7}$$

Plus the following formulas for linking parameters β_{1i} and β_{2i} as described in definition/notation of previous section:

$$K_{\pi i} = \left(1+\beta_{1i}\right)K_i \tag{8}$$

$$C_{\pi i} = \left(1+\beta_{2i}\right)C_i \tag{9}$$

TC(T_π) – total system cost per cycle contains the following: in-house setup and variable costs, fixed and variable outsourcing expenses, variable disposal cost for scraps, and holding costs for finished and defective products in a replenishing cycle (see Eq. (10)).

$$TC\left(T_\pi\right) = \sum_{i=1}^{L}\left\{\begin{array}{l} K_i + C_i\left(1-\pi_i\right)Q_i + K_{\pi i} + C_{\pi i}\left(\pi_i Q_i\right) \\ +C_{\mathrm{S}i} x_i\left[\left(1-\pi_i\right)Q_i\right] \\ +h_i\left[\frac{H_{1i} + d_{1i}t_{1i\pi}}{2}\left(t_{1i\pi}\right) + \frac{H_i}{2}\left(t_{2i\pi}\right)\right] \end{array}\right\} \tag{10}$$

Substitute Eqs. (8) and (9) in Eq. (10), *TC*(T_π) becomes as follows:

$$TC\left(T_\pi\right) = \sum_{i=1}^{L}\left\{\begin{array}{l} K_i + C_i\left(1-\pi_i\right)Q_i + \left(1+\beta_{1i}\right)K_i \\ +\left(1+\beta_{2i}\right)C_i\left(\pi_i Q_i\right) \\ +C_{\mathrm{S}i} x_i\left[\left(1-\pi_i\right)Q_i\right] \\ +h_i\left[\frac{H_{1i} + d_{1i}t_{1i\pi}}{2}\left(t_{1i\pi}\right) + \frac{H_i}{2}\left(t_{2i\pi}\right)\right] \end{array}\right\} \tag{11}$$

Replace Eq. (1) in Eq. (11), and employ $E[x_i]$ and $E[T_\pi]$ to deal with effects of randomness of x_i on the expected cost $E[TCU(T_\pi)]$, with extra derivation one has the following:

$$\begin{aligned} E\left[TCU\left(T_\pi\right)\right] &= \frac{E\left[TC\left(T_\pi\right)\right]}{E\left[T_\pi\right]} \\ &= \sum_{i=1}^{L}\left\{\frac{K_i\left(1+\beta_{1i}\right)}{T_\pi} + \frac{K_i}{T_\pi} + \frac{h_i\lambda_i T_\pi}{2}\right\} \\ &+ \sum_{i=1}^{L}\frac{\lambda_i}{E_{1i}}\left\{\begin{array}{l}\left(1+\beta_{2i}\right)C_{\mathrm{1}}\pi_i + C_i\left(1-\pi_i\right) + C_{\mathrm{S}i}E\left[x_i\right]\left(1-\pi_i\right) \\ -\frac{h_i\lambda_i T_\pi\left(1-\pi_i\right)}{2E_{1i}P_{1i}}\left[\left(1+\pi_i\right) - 2E\left[x_i\right]\left(1-\pi_i\right)\right]\end{array}\right\} \end{aligned}$$

(12)

where $E_{1i} = \left[1 - E\left[x_i\right]\left(1-\pi_i\right)\right]$

3.1. The optimal common cycle time

The following are the first- and second-derivative of the expected annual system cost $E[TCU(T_\pi)]$:

$$\begin{aligned} \frac{dE\left[TCU\left(T_\pi\right)\right]}{dT_\pi} &= \sum_{i=1}^{L}\left\{\frac{-K_i\left(1+\beta_{1i}\right)}{T_\pi^2} - \frac{K_i}{T_\pi^2} + \frac{h_i\lambda_i}{2}\right\} \\ &+ \sum_{i=1}^{L}\frac{\lambda_i}{E_{1i}}\left\{-\frac{h_i\lambda_i\left(1-\pi_i\right)}{2E_{1i}P_{1i}}\left[\left(1+\pi_i\right) - 2E\left[x_i\right]\left(1-\pi_i\right)\right]\right\} \end{aligned} \tag{13}$$

$$\frac{d^2E\left[TCU\left(T_\pi\right)\right]}{dT_\pi^{\;2}} = \sum_{i=1}^{L}2\left[\frac{K_i\left(1+\beta_{1i}\right) + K_i}{T_\pi^3}\right] \tag{14}$$

Since $(1+\beta_{1i})$, T_π, and K_i are all positive, hence, the second-derivative of $E[TCU(T_\pi)]$ (i.e., Eq. (14)) is positive. Therefore, $E[TCU(T_\pi)]$ is convex for all T_π different from zero. To locate the optimal T_π, we set the first-derivative of $E[TCU(T_\pi)] = 0$ and solve T_π^* as follows:

$$\begin{aligned} \frac{dE\left[TCU\left(T_\pi\right)\right]}{dT_\pi} &= \sum_{i=1}^{L}\left\{\frac{-K_i\left(1+\beta_{1i}\right)}{T_\pi^2} - \frac{K_i}{T_\pi^2} + \frac{h_i\lambda_i}{2}\right\} \\ &+ \sum_{i=1}^{L}\frac{\lambda_i}{E_{1i}}\left\{-\frac{h_i\lambda_i\left(1-\pi_i\right)}{2E_{1i}P_{1i}}\left[\left(1+\pi_i\right) - 2E\left[x_i\right]\left(1-\pi_i\right)\right]\right\} = 0 \end{aligned} \tag{15}$$

With extra derivations, the following T_π^* is found.

$$T_\pi^* = \sqrt{\frac{2\sum_{i=1}^{L}\left[K_i\left(2+\beta_{1i}\right)\right]}{\sum_{i=1}^{L}\left\{h_i\lambda_i - \frac{h_i\lambda_i^2\left(1-\pi_i\right)}{\left[1-E\left[x_i\right]\left(1-\pi_i\right)\right]^2 P_{1i}}\left[\left(1+\pi_i\right) - 2E\left[x_i\right]\left(1-\pi_i\right)\right]\right\}}} \tag{16}$$

4. Numerical illustration

This section offers a numerical illustration to confirm that our research outcomes can be practically used.

Consider five distinct product end items that need to be manufactured by a multi-item single-machine production system with an outsourcer and random defective rate. Assumption of values of parameters for each item (see Eqs. (8) and (9)) and for the proposed manufacturing system is given in Table 1.

For average outsourcing factor $\overline{\pi}$ at 0.4, by calculating of Eqs. (16) and (12), one can find the optimal common manufacturing cycle time $T_\pi{}^*$ = 0.6829 years and E[$TCU(T_\pi{}^*)$] = $2,271,218. Analytical outcome on the effect of variations in common manufacturing time T_π on E[$TCU(T_\pi{}^*)$] is portrayed in Figure 3.

4.1. The impact from variations in $\overline{\pi}$

Applying the formulas of the proposed model (see section 3), we can further reveal the influence of variations in $\overline{\pi}$ on diverse system variables as exhibited in Table 2. The detailed cost components that contribute to E[$TCU(T_\pi{}^*)$] can also be revealed from the proposed model as illustrated in Figure 4 (for $\overline{\pi}$= 0.4). It indicates the outsourcing related cost is 42% of E[$TCU(T_\pi{}^*)$] (where 1.1% is from setup and 40.9% is from variable outsourcing cost; see Table 2); and quality cost due to scrap is $83,373 or 3.67% of E[$TCU(T_\pi{}^*)$] (see also Table 2).

Further exploration reveals the effect of variations in $\overline{\pi}$ on each end item's total cost, as depicted in Figure 5. It is noted that since outsourcing cost is more expensive than in-house manufacturing cost, so as $\overline{\pi}$ increases, the total cost for each end item goes up accordingly.

Table 1. Assumption of values of parameters for each item and for the proposed system.

Item #	π_i	K_i	β_{1i}	$K_{\pi i}$	C_i	β_{2i}	$C_{\pi i}$	x_i	C_{Si}	h_i	P_{1i}	λ_i
1	0.4	10000	-0.60	4000	80	0.40	112.0	5%	20	10	58000	3000
2	0.4	11000	-0.65	3850	90	0.35	121.5	10%	25	15	59000	3200
3	0.4	12000	-0.70	3600	100	0.30	130.0	15%	30	20	60000	3400
4	0.4	13000	-0.75	3250	110	0.25	137.5	20%	35	25	61000	3600
5	0.4	14000	-0.80	2800	120	0.20	144.0	25%	40	30	62000	3800

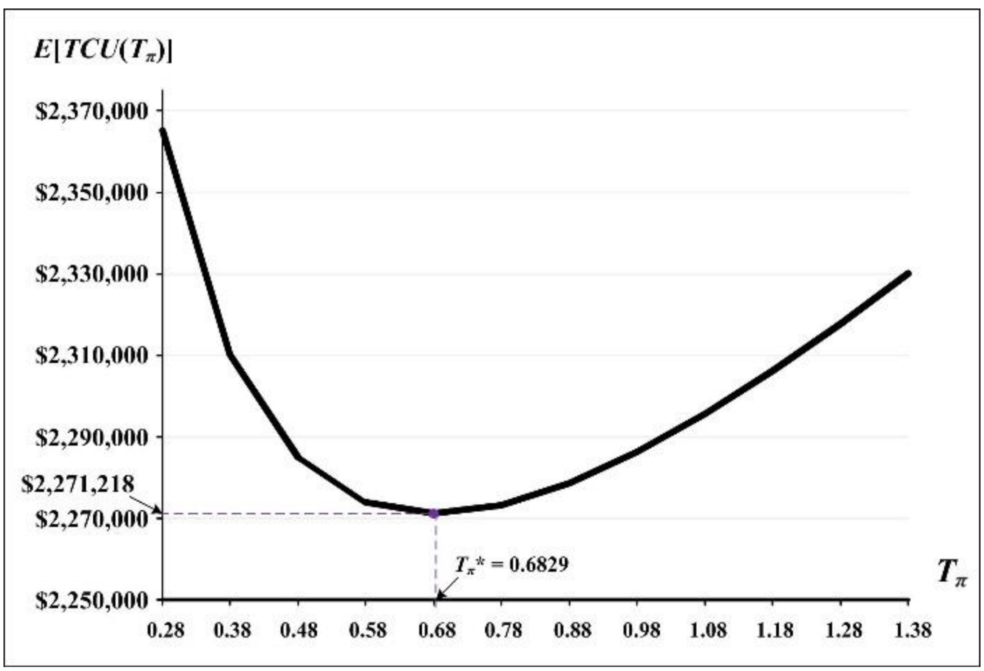

Figure 3. Effect of variations in common manufacturing time Tπ on E[TCU(Tπ*)].

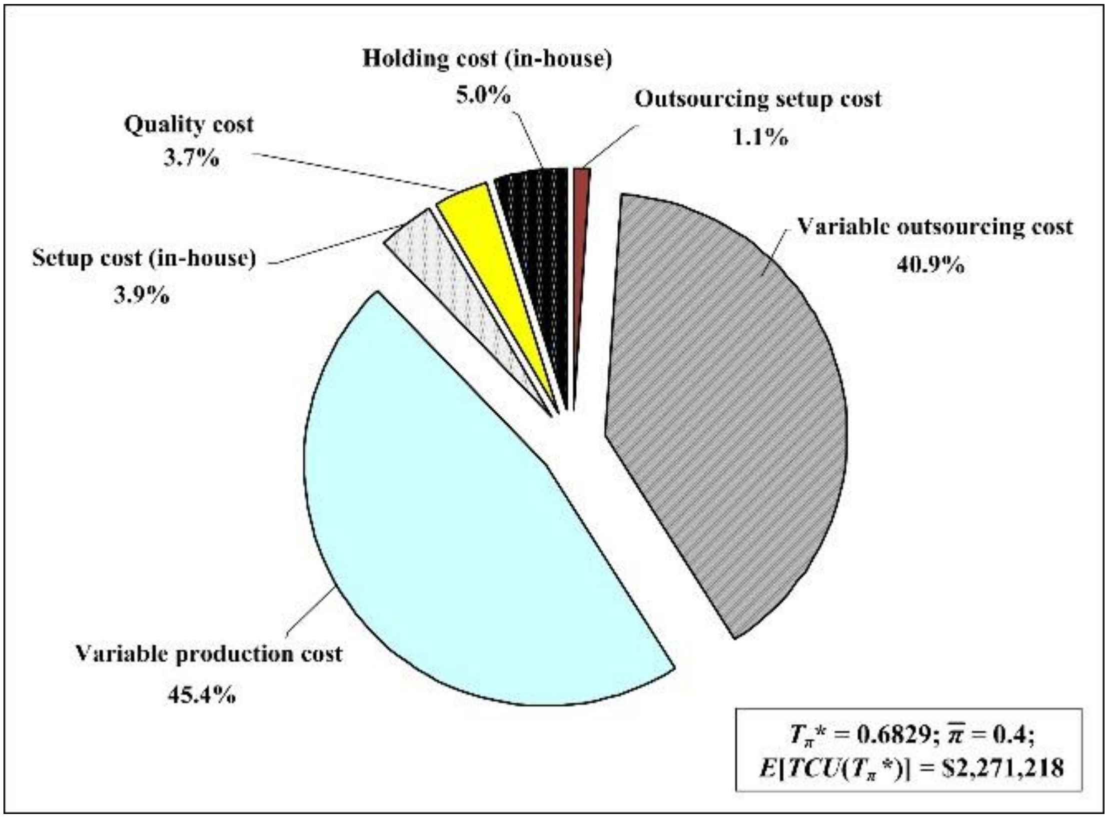

Figure 4. The detailed cost components of E[TCU(Tπ*)].

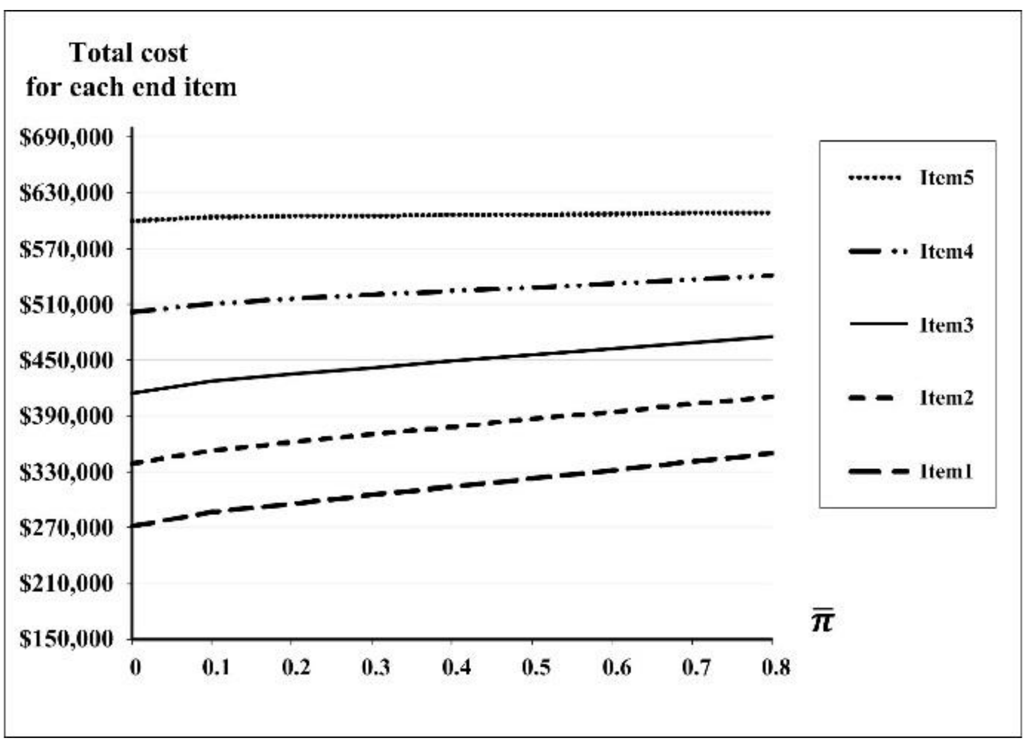

Figure 5. Effect of variations in $\overline{\pi}$ on total cost for each

Table 2. Influence of variations in $\overline{\pi}$ on diverse system variables.

$\overline{\pi}$	T_π^*	$E[TCU(T_\pi^*)]$ [A]	% change	Outsourcing related cost [B]	% [B]/[A]	Total in-house related costs [C]	% [C]/[A]	Sum of uptimes	Machine utilization	Quality cost alone ($\overline{x}$=15%) [D]	% [D]/[A]
0.00	0.6854	$2,128,365	-	$0	0.0%	$2,128,365	100.0%	0.2104	30.7%	$209,345	9.84%
0.05	0.6856	$2,168,705	1.9%	$145,333	6.70%	$2,023,372	93.3%	0.1991	29.0%	$190,166	8.77%
0.10	0.6856	$2,183,474	2.6%	$264,061	12.1%	$1,919,413	87.9%	0.1878	27.4%	$172,002	7.88%
0.15	0.6855	$2,198,202	3.3%	$381,731	17.4%	$1,816,471	82.6%	0.1766	25.8%	$154,837	7.04%
0.20	0.6853	$2,212,889	4.0%	$498,360	22.5%	$1,714,528	77.5%	0.1655	24.1%	$138,652	6.27%
0.25	0.6849	$2,227,534	4.7%	$613,966	27.6%	$1,613,568	72.4%	0.1544	22.5%	$123,430	5.54%
0.30	0.6843	$2,242,138	5.4%	$728,565	32.5%	$1,513,574	67.5%	0.1434	21.0%	$109,154	4.87%
0.35	0.6837	$2,256,699	6.0%	$842,171	37.3%	$1,414,529	62.7%	0.1324	19.4%	$95,807	4.25%
0.40	0.6829	$2,271,218	6.7%	$954,800	42.0%	$1,316,418	58.0%	0.1216	17.8%	$83,373	3.67%
0.45	0.6820	$2,285,692	7.4%	$1,066,468	46.7%	$1,219,225	53.3%	0.1109	16.3%	$71,836	3.14%
0.50	0.6809	$2,300,123	8.1%	$1,177,188	51.2%	$1,122,935	48.8%	0.1002	14.7%	$61,182	2.66%
0.55	0.6798	$2,314,509	8.8%	$1,286,976	55.6%	$1,027,534	44.4%	0.0897	13.2%	$51,394	2.22%
0.60	0.6786	$2,328,851	9.4%	$1,395,844	59.9%	$933,006	40.1%	0.0793	11.7%	$42,458	1.82%
0.65	0.6772	$2,343,147	10.1%	$1,503,808	64.2%	$839,339	35.8%	0.0689	10.2%	$34,360	1.47%
0.681	0.6763	$2,351,987	10.5%	$1,570,297	66.8%	$781,691	33.2%	0.0626	9.3%	$29,754	1.27%
0.70	0.6758	$2,357,397	10.8%	$1,610,880	68.3%	$746,518	31.7%	0.0587	8.7%	$27,086	1.15%
0.75	0.6743	$2,371,601	11.4%	$1,717,073	72.4%	$654,529	27.6%	0.0486	7.2%	$20,623	0.87%
0.80	0.6727	$2,385,759	12.1%	$1,822,400	76.4%	$563,359	23.6%	0.0387	5.8%	$14,956	0.63%
0.85	0.6710	$2,399,869	12.8%	$1,926,874	80.3%	$472,996	19.7%	0.0288	4.3%	$10,072	0.42%
0.90	0.6692	$2,413,933	13.4%	$2,030,507	84.1%	$383,426	15.9%	0.0191	2.9%	$5,960	0.25%
0.95	0.6674	$2,427,949	14.1%	$2,133,310	87.9%	$294,638	12.1%	0.0095	1.4%	$2,607	0.11%
1.00	0.6655	$2,351,755	10.5%	$2,235,297	95.1%	$116,458	4.9%	0	0%	$0	0%

On the other hand, Figure 6 depicts the effect of changes in $\overline{\pi}$ on total scrap cost in the system. It indicates that as $\overline{\pi}$ increases, the portion of in-house production quantities decreases, so does the total scrap costs in the system.

Besides, a benefit is gained regarding the decline in machine utilization. Figure 7 illustrates the effect of variations in $\overline{\pi}$ on machine utilization. It indicates that as $\overline{\pi}$ rises, machine utilization considerably declines, and at $\overline{\pi}$= 0.4, utilization goes down to 17.8%. This reduction in utilization is at the cost of 6.7% increase of $E[TCU(T_\pi^*)]$ (see Table 2, $E[TCU(T_\pi^*)]$ raises from $2,128,365 to $2,271,218).

The analytical result also reveals a piece of important information on $\overline{\pi}$ ratio to help make-or-buy decision making. That is as $\overline{\pi}$ increase to 0.681, $E[TCU(T_\pi^*)]$ = $2,351,987 (see Table 2), which is greater than $2,351,755 for $\overline{\pi}$= 1. It suggests that for any $\overline{\pi}$ > 0.681, the 100% outsourcing policy is in favor in terms of cost savings (see Table 2).

4.2. The effect from the difference in $\bar{x}$

Another key factor in our study is the average scrap rate $\overline{x}$ of in-house processes. Figure 8 exhibits the impact of variations in $\overline{x}$ on the total cost for each end item. It shows that the total costs increase as $\overline{x}$ raises, for the scrap cost boost up.

Also, the influence of changes in $\overline{x}$ on various system cost components is explored and its outcome is shown in Figure 9. It is noted that the quality cost surges considerably, as $\overline{x}$ rises.

Figure 10 depicts the analytical result of important information on $\overline{x}$ ratio to help make-or-buy decision making. In our example, at $\overline{\pi}$ = 0.4, it shows that as $\overline{x}$ increases to 0.263 and up, a 100% outsourcing policy (i.e., a 'buy' decision) is in favor in terms of cost savings. Further study points out that at $\overline{\pi}$ = 0, if the in-house production has scrap rate $\overline{x}$ >= 0.323 (this is slightly higher previous 0.263 (when $\overline{\pi}$ = 0.4)), the buy decision is in favor.

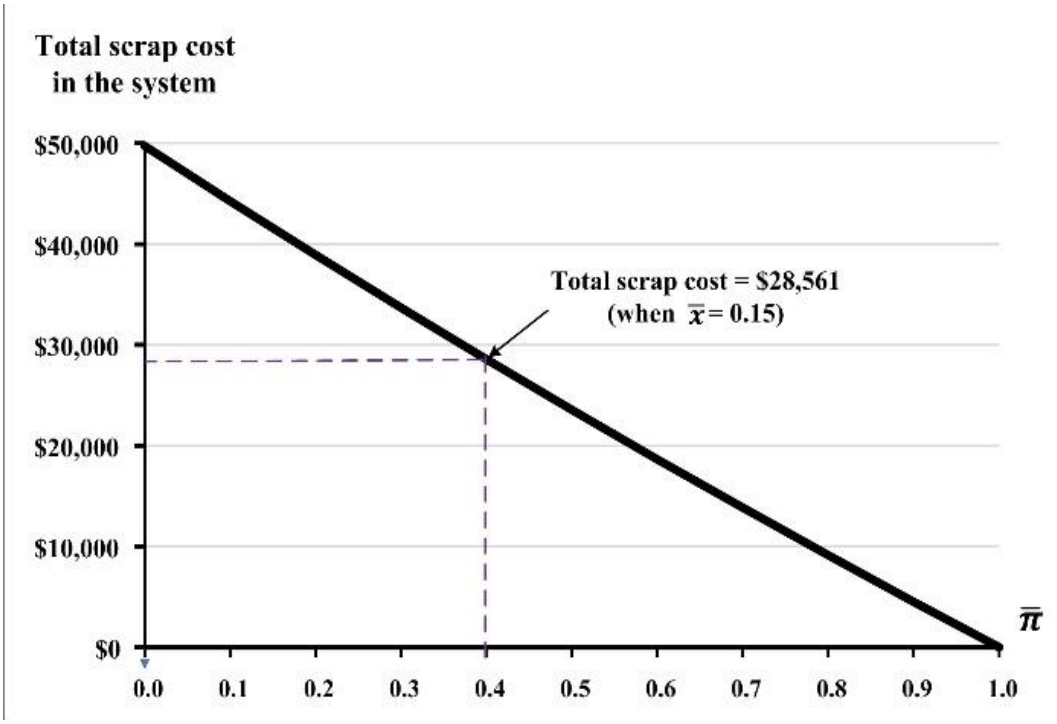

Figure 6. Impact of differences in $\overline{\pi}$ on total scrap cost in the system.

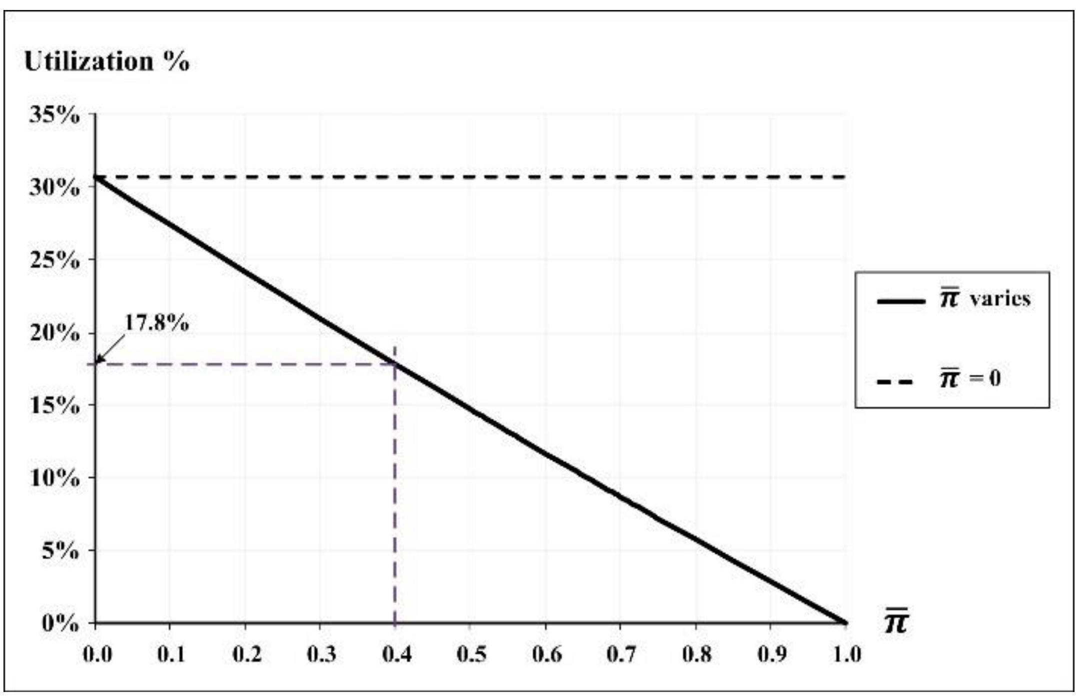

Figure 7. The effect of variations in $\overline{\pi}$ on machine utilization.

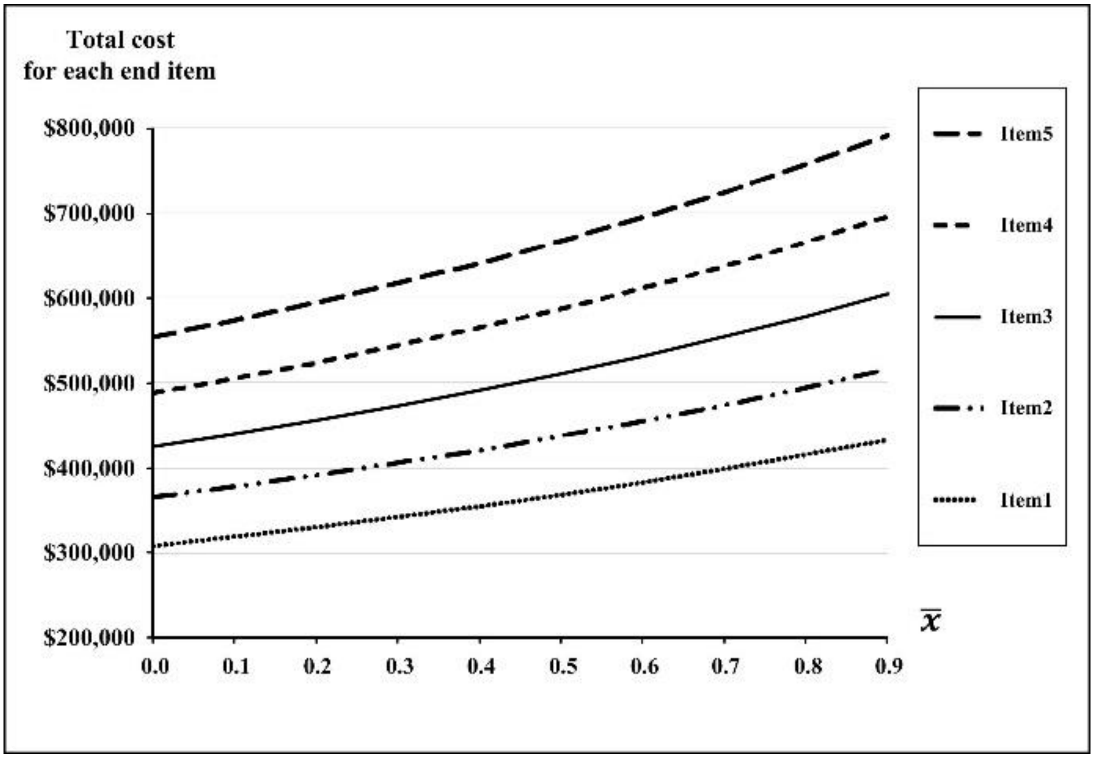

Figure 8. The effect of differences in $\bar{x}$ on total cost for each end item.

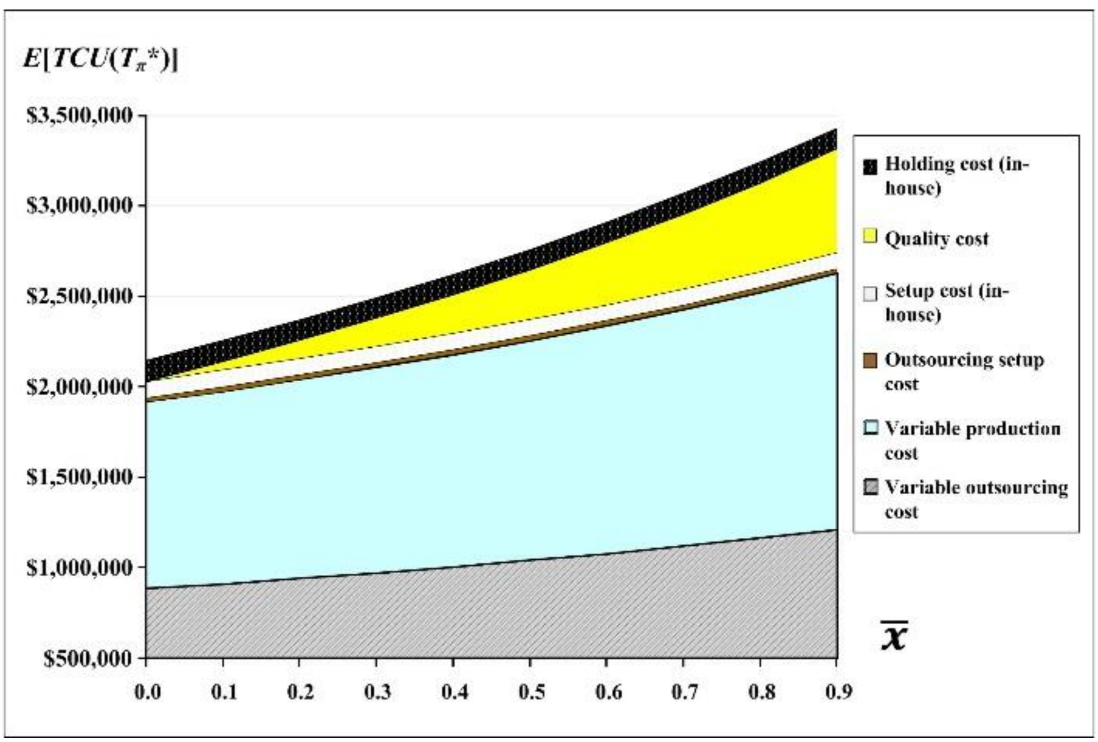

Figure 9. The influence of changes in $\bar{x}$ on various system cost components.

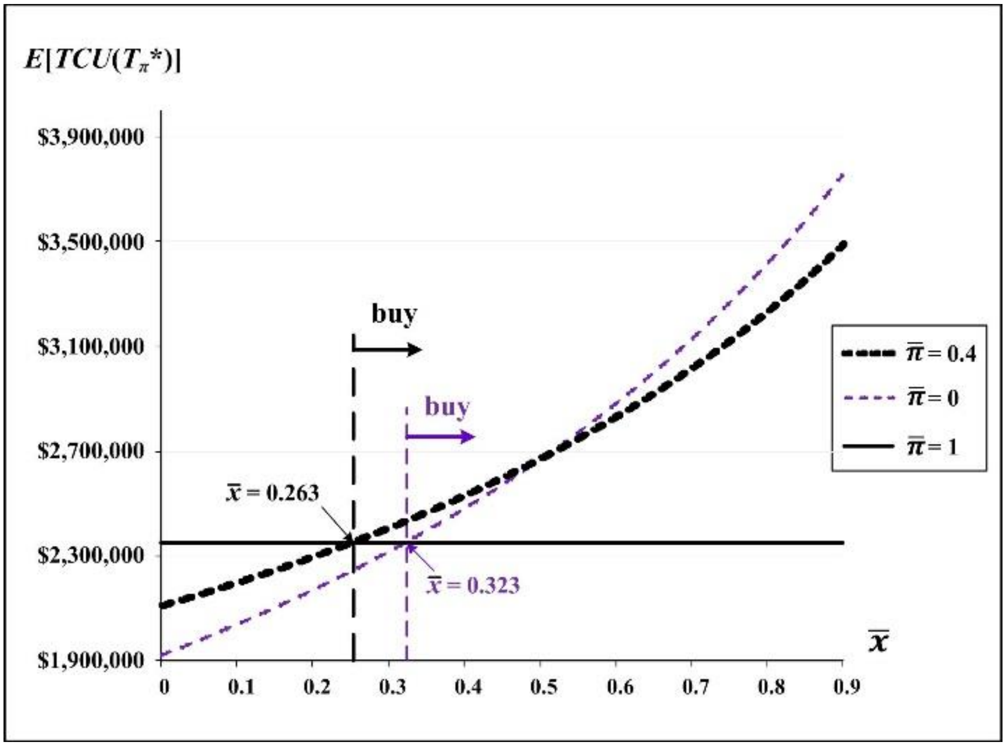

Figure 10. The impact of differences in $\bar{x}$ on E[TCU(Tπ*)] to help the make-or-buy decision making.

4.3. Joint impacts from diverse system factors

The collective influences of changes in the average outsourcing setup cost linking parameter $\overline{\beta_1}$ and outsourcing factor $\overline{\pi}$ on the optimal T_π* is analyzed and the results are depicted in Figure 11. It shows that as $\overline{\beta_1}$ increases, T_π* raises significantly, and T_π* declines slightly as $\overline{\pi}$ goes up.

Figure 12 shows the joint impacts of variations in T_π and average unit outsourcing cost linking parameter $\overline{\beta_2}$ on E[TCU(T_π)]. It indicates that as T_π moves up and away from its optimal point 0.6829, E[TCU(T_π)] increases notably; and as $\overline{\beta_2}$ goes up, E[TCU(T_π)] increases radically.

Moreover, the analytical results also reveals important information on $\overline{\beta_2}$ ratio to help managerial decision making (see Figure 13). It shows that for $\overline{\pi}$ = 0.4 (as in our example), if $\overline{\beta_2}$ is less than or equal to 0.103, the management should consider implementing this 40% partial outsourcing policy, for it not only can reduce expected annual system cost but also lowering in-house machine utilization (see Figures 13 and 7).

Additionally, our model can provide the critical $\overline{\beta_2}$ ratio for any given $\overline{\pi}$, for example, for an in-house average defective rate at $\overline{x}$ = 0.15, if $\overline{\beta_2}$ is less than or equal to 0.171, a 100% outsourcing policy is in favor (for details please refer to Figure 13, the lines of $\overline{\pi}$ = 0 and $\overline{\pi}$ = 1).

4.4. The managerial insights from the study's results

From the above research's results, the production planners/managers can disclose the impact of an outsourcing strategy and random scraps on the optimal replenishing policy (in terms of rotation cycle length) of a multi-item manufacturing system. These include the individual and collective influence of variations in average: (i) outsourcing factor, (ii) scrap rate, and (iii) outsourcing-relevant costs on the optimal system's rotation cycle time, total costs, and overall machine utilization, etc., to facilitate managerial operational controlling and decision making.

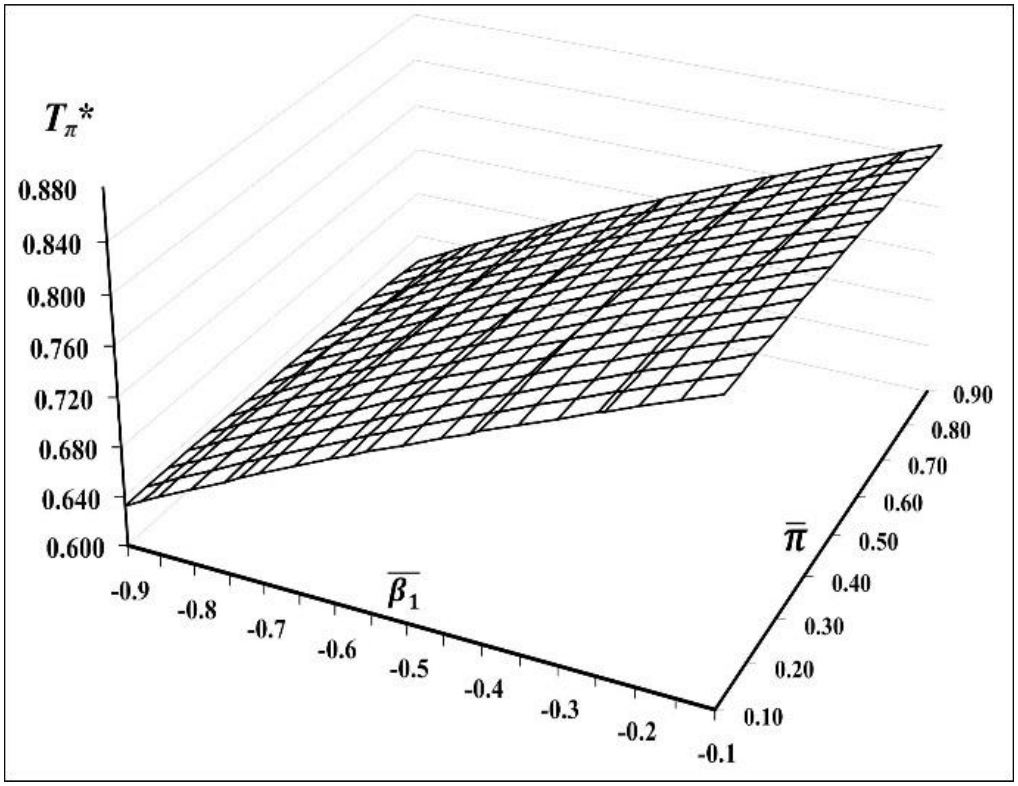

Figure 11. Collective influences of changes in $\overline{\beta_1}$ and $\overline{\pi}$ on T_π*.

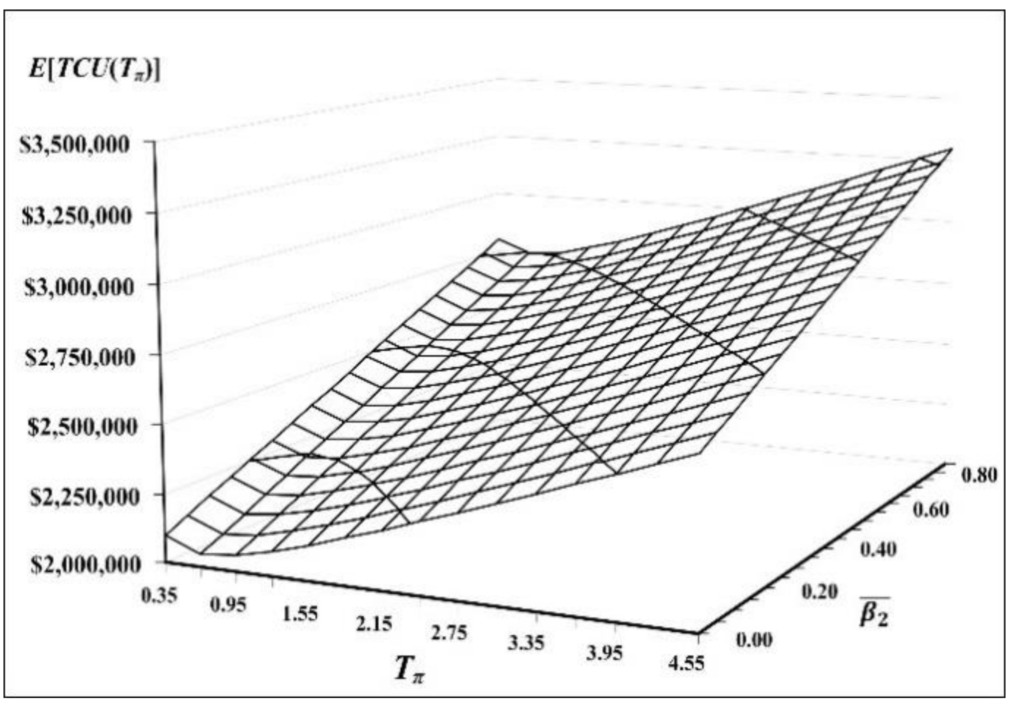

Figure 12. Joint effects of variations in T_π and $\overline{\beta_2}$ on E[$TCU(T_\pi$*)]

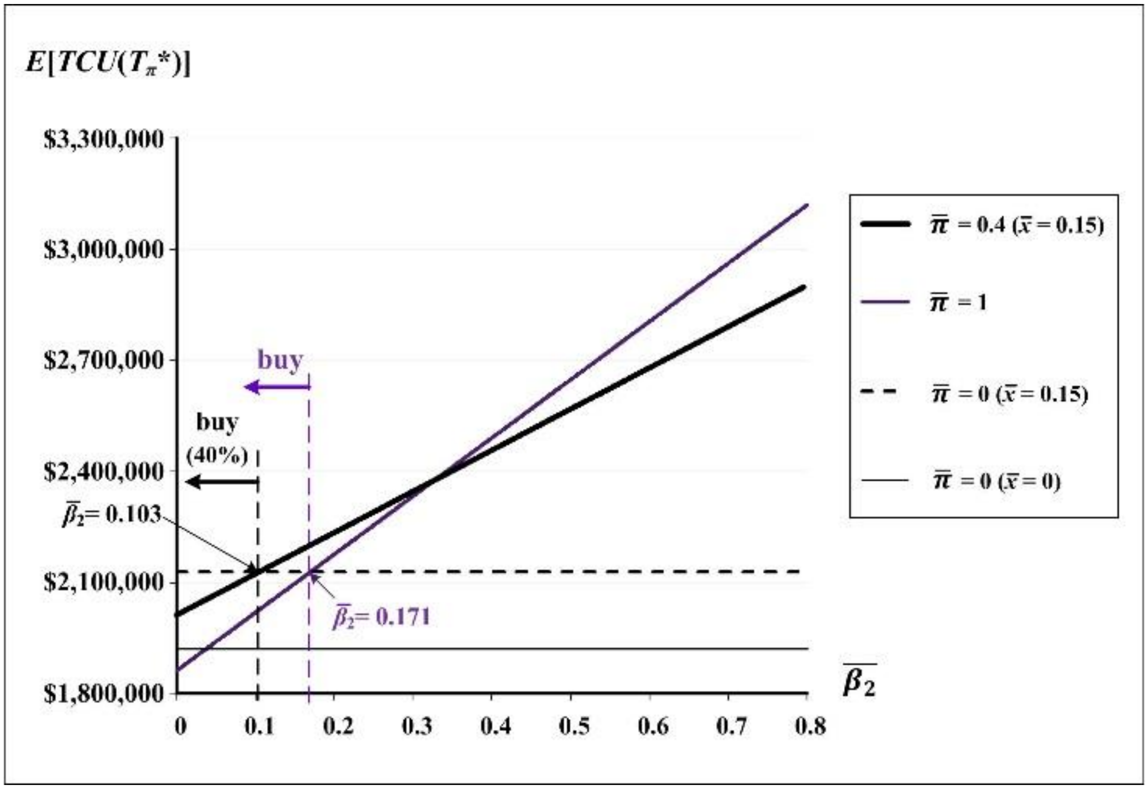

Figure 13. Impact of differences in $\overline{\beta_2}$ on $E[TCU(T_\pi^*)]$ to help the make-or-buy decision making.

5. Concluding remarks

This study develops a mathematical model representing a multi-item single-machine hybrid stock refilling system with random defective items. We employ mathematical derivations to find long-run average system expenses and uses an optimization technique to derive a closed-form common rotation time for this hybrid system. This study's main contribution includes (1) Establishing a decision support model for production managers/planners to deal with the specific multi-item single-machine hybrid refilling system with random scrap. (2) Disclosure of crucial individual/collective impacts of variations in outsourcing percentages/ relevant-cost and scrap rates on the optimal replenishing policy, total system cost, overall utilization, and each system's cost-component (refer to Figures 3 to 13). These findings can facilitate the planning, controlling, and decision making of such a hybrid system. Finally, an important topic for future study is investigating the effect of random multiproduct demand rates on the problem.

Acknowledgments

Authors are grateful for sponsorship from the Ministry of Science and Technology of Taiwan (under project #: MOST 105-2410-H-324-003).

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