Journal of Applied Research and Technology

www.jart.icat.unam.mx

Journal of Applied Research and Technology 20 (2022) 418-429

Original

Rotation cycle time and delivery decision for a multi-item producer-retailer integrated system featuring overtime and random scrap

Hong-Dar Lin[a] • Yi-Jing Huang[a] • Victoria Chiu[b] • Yuan-Shyi Peter Chiu[a]*

[a]*Dept. of Industrial Engineering and Management, The Chaoyang University of Technology, Taichung, Taiwan 413*
[b]*Dept. of Accounting, Finance and Law, The State University of New York at Oswego, NY 13126, USA*

Received 06 24 2020; accepted 05 27 2022
Available 08 31 2022

Abstract: A multi-item producer-retailer integrated system featuring overtime and random scrap is studied. The objectives are to jointly decide the most economic rotation fabrication cycle time and distribution of products. In order to meet the increasing demands of diversified end items, production managers today need to plan a multiproduct fabrication schedule and to expedite both manufacturing and transportation times, so that they can meet product demands as quickly as possible. Also, due to potential uncontrollable reasons, scrap items are generated randomly in a real fabrication process. To address the aforementioned issues, this study examines a multi-item producer-retailer integrated system featuring overtime and random scrap. We build a mathematical model to interpret the proposed multi-item producer-retailer integrated system which incorporates shipping and retailer's holding cost. The Hessian matrix equations are used for solving the optimality of the system. Diverse important system information can now be exposed to backing managerial decision makings, which includes individual and combined influences of variations in particular system factor(s) (such as scrap rate and overtime related parameters) on the specific system performance.

Keywords: Production management; multiproduct inventory system; overtime; rotation cycle time; scrap; producer-retailer integrated system

*Corresponding author.
E-mail address: ypchiu@cyut.edu.tw (Yuan-Shyi Peter Chiu).
Peer Review under the responsibility of Universidad Nacional Autónoma de México.

1. Introduction

This work intends to jointly decide the frequency of delivery and rotation cycle time for a multiproduct producer-retailer integrated system featuring overtime and scrap. To meet the increasing demands on diversity of end items, production managers today constantly require a multiproduct fabrication plan. Dixon and Silver (1981) examined an optimal lot-size problem with one work center that produce multiproduct. The known period by period demands for each item were assumed for a finite period of time. Setup costs were fixed and the fabrication and holding costs were assumed to be linear. These costs were varied for different products. Their goal is to decide batch size so that (i) total costs can be kept minimum; (ii) no shortage occurs; and (iii) capacity does not exceed. A heuristic was proposed to decide the feasible solution, and examples were used to show that their heuristic in most circumstances can provide good solution with a relatively short computer time. Mitchell (1988) proposed an algorithm based on the generalized Knapsack duality algorithm to study a multi-item inventory system with service level constraint. The objective was to locate the system's approximate optimal policies. As a result, it pointed out that by applying the proposed model, the operating expense can be considerably cut down as compared to the existing uniform service model. Sambasivan and Schmidt (2002) solved the multi-plant, multiproduct, capacitated lot-size problems with inter-plant transfers. To tackle the problems, they proposed a heuristic procedure starting with solution to un-capacitated problem. They further used a smoothing routine to eliminate any violations of capacity. Broad experimentations were performed to verify accurate of their heuristic results, which were executed on IBM mainframe environments. Sancak and Salmann (2011) determined the optimal purchasing and inbound receiving policies for a producer that acquires multi-item from a provider. Their objective was to meet the needs in fabrication plan over a finite period of times, and to lower the total delivery and stock holding expenses. The delivery cost per truck is charged to the producer. They proposed an idea of shipping a full truckload as needed and used safety stocks to cover those requirements that are less than a full truckload. The influences of such a delay shipping on the service levels and related cost were analyzed. It pointed out that the proposed idea considerably cut down the delivery and stock holding costs. Chiu et al. (2016) investigated a two-machine multiproduct finite production rate model considering delayed differentiation, scrap, and multi-shipment policy. A cluster of multiproduct that shares a component and two-phase fabrication procedures are assumed. In phase 1 only the mutual components are produced by machine 1; while in phase 2, a separate machine fabricates the diverse end products. The objectives were to simultaneously reduce manufacturing time and overall system costs. Their results were compared to both single-phase and two-phase single-machine schemes, to demonstrate the merits of their proposed scheme. Extra articles that explored diverse aspects of multiproduct replenishment systems and/or supply chains can be found elsewhere (Barata, 2021; Benítez, 2019; Chiu et al., 2020; Chiu, S. et al., 2021; Farmand et al., 202; Kumar et al., 2019; Lin et al., 2019; López-Ruíz, & Carmona- Pourmohammadi et al., 2020; Raza & Govindaluri, 2019).

Random defects often exist in real fabrication processes due to different uncontrollable causes. Eroglu and Ozdemir (2007) studied an economic order quantity (EOQ) model featuring defects and backlogging. All items are screened to identify defects and scraps from perfect goods. Impact of defect rate on EOQ was investigated. Moussawi-Haidar et al. (2013) examined an EOQ system with unreliable supplier. Acceptance sampling policy was used toward every incoming order to determine whether a follow-up 100% inspection is required or not (it is required only if outcomes of quantity of imperfect goods in sampling plan exceeds the acceptable standard). Non-linear math program is formulated that combines the stock refilling and quality issues into a profit model in order to simultaneously decide the best lot size and sampling plan, which help achieve the desirable average outgoing quality limit. A procedure along with numerical illustration was provided to jointly calculate the optimal lot size and sampling plan to the problem and demonstrate applicability and performance of their results. Additional studies (Afshar-Nadjafi et al., 2019; Bhagat et al., 2021; Chiu, Lin, & Wu, 2020; Daryanto, & Christata, 2021; Hariga & Ben-Daya, 1998; Lesmono et al., 2020; Mokao, 2020; Sana, 2010; Terdpaopong et al., 2021; Yamada et al., 2021) also focused on different features of imperfection products and/or manufacturing systems.

Further, an overtime option is usually treated as an effective means to expedite fabrication processes/time, so as the demands can be met sooner. Makino and Tominaga (1995) indicated that the estimation of periodic fabrication quantity (e.g. by month or by year) is required initially to plan production of a new product. Secondly, to design and construct a standardized flexible assembly system which includes numerous sub-systems (such as feeding, moving work-in- process, assembly, etc.) is necessary for meeting the required cycle time. Accordingly, they estimated and discussed both fabrication rate and the required cycle length for a classic flexible assembly system. El-Gohary et al. (2009) applied the optimal control theory to adjust the fabrication rate for a deteriorating inventory system, wherein a manufacturing firm makes certain items at a constant rate and is intended to improve its fabrication rate. An optimal control model was constructed/formulated to deal with the problem, and as a result, an explicit solution along with illustrative example was presented to show applicability of the obtained results. Extra papers (Campbell, 2017; Chiu et al., 2020; Chiu, Y. et al., 202; Chiu, Lin, & Wu, 2020; Ivanov et al., 2019; Lin et al., 2019; Lesmono et al., 2020; Mokao, 2020; Nicolaisen, 2011; Ohmori & Yoshimoto, 2021;) also addressed systems with overtime options and adjusted fabrication/output rates. This study intends to jointly decide the rotation cycle length and frequency of delivery for a multiproduct producer-retailer integrated system featuring overtime and random scrap.

2. The multiproduct producer-retailer integrated system

2.1. List of notation

L= total number of different products to be produced in the proposed study,
λ_i = annual demand of product i (where i equals to 1, 2, …, L),
Q_i= lot size of product i,
T_A= rotation cycle time – decision variable,
n= number of deliveries per cycle – a decision variable,
P_{1iA}= annual manufacturing rate of product i when overtime is implemented,
P_{1i}=standard annual manufacturing rate of product i without overtime option,
α_{1i}=added proportion of production rate due to overtime implementation (where $\alpha_{1i} > 0$),
C_{iA}=unit production cost of product i when overtime is implemented,
C_i=standard unit cost of product i without overtime option,
α_{3i}= connecting factor between C_i and C_{iA} (where $\boldsymbol{\alpha}_{3i} > 0$),
K_{iA}=setup cost of product i when overtime is implemented,
K_i=standard setup cost of product i without overtime option,
α_{2i}=connecting factor between costs of K_i and K_{iA} (where $\alpha_{2i} > 0$),
t_{1iA}=production uptime of product i,
t_{2iA}=delivery time of product i,
h_i=unit holding cost of product i,
h_{2i}=unit holding cost of product i at the retailer's side,
x_i=random scrap rate of product i fabricated in production process,
$E[x_i]$ =the expected scrap rate of product i,
d_{1iA}=manufacturing rate of scrap product i during uptime,
C_{Si}= unit disposal cost of product i,
H_i=stock level of perfect product i at the end of its uptime,
K_{1i}=fixed shipping cost of product i,
C_{Ti}=unit delivery cost of product i,
t_{niA}=fixed time interval between two successive deliveries of product i,
T=rotation cycle time in the same system without overtime,
t_{1i}=uptime of product i in the same system without overtime,
t_{2i}=delivery time of product i in the same system without overtime,
d_{1i}=production rate of scrapped product i in the same system without overtime option,
$I(t)_i$=level of end product i at time t,
$I_D(t)_i$=level of scrapped product i,
$I_C(t)_i$=stock level of product i at retailer side,
$E[T_A]$=the expected cycle time,
$TC(T_A, n)$ = total system cost per cycle,
$E[TCU(T_A, n)]$ = the expected system cost per unit time,

$\overline{P_{1A}}$ = the average of $\boldsymbol{P_{1iA}}$,
$\overline{P_1}$ = the average of $\boldsymbol{P_{1i}}$,
$\overline{x}$ = the average of $\boldsymbol{x_i}$,
$\overline{C_A}$ = the average of $\boldsymbol{C_{iA}}$,
$\overline{C}$ = the average of $\boldsymbol{C_i}$,
$\overline{\alpha_1}$ = the average of $\boldsymbol{\alpha_{1i}}$,
$\overline{\alpha_3}$ = the average of $\boldsymbol{\alpha_{3i}}$.

2.2. Assumption and formulation

This study explores rotation cycle time and delivery decision for a multi-item producer-retailer integrated system featuring overtime and random scrap. With the aim of reducing production time overtime option is often adopted along with routine manufacturing plan. Consider annual demands λ_i of L different products must be satisfied by a fabrication system using a rotation cycle rule, that is, during a production cycle each product is replenished one time, in sequence (Figure 1) at an expedited rate P_{1iA} (i.e., overtime is incorporated into the production rate).

The production processes are not perfect, x_i proportion of scraps (where i = 1, 2, …, L) are produced at annual rate d_{1iA} (Figure 2). Due to overtime policy, the following assumptions are made in this study:

$$P_{1iA} = (1 + \alpha_{1i})P_{1i} \tag{1}$$
$$K_{iA} = (1 + \alpha_{2i})K_i \tag{2}$$
$$C_{iA} = (1 + \alpha_{3i})C_i \tag{3}$$

where P_{1i}, K_i, and C_i represent the standard production rate, setup and unit production costs; and α_{1i} , α_{2i}, and α_{3i} are the connecting factors between expeditious and standard system variables (see notation list). A few equations can be clearly seen from Figures 1 and 2 as follows:

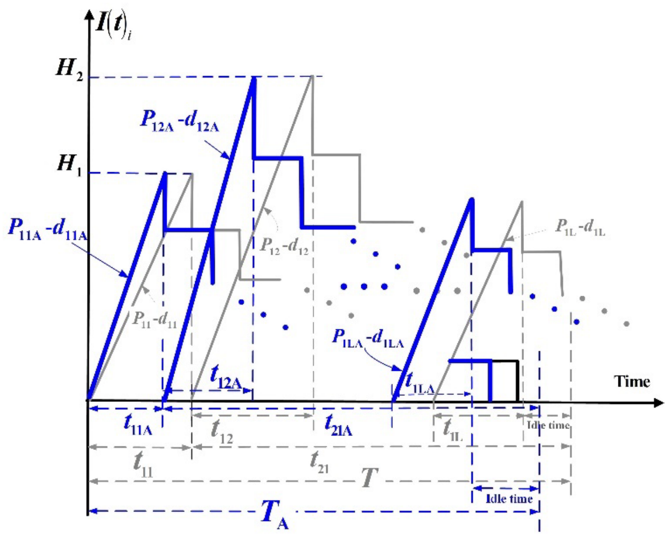

Figure 1. Inventory status of perfect product *i* in this multi-item producer-retailer integrated system featuring overtime and random scrap (in blue) in comparison with the same system without overtime (in grey).

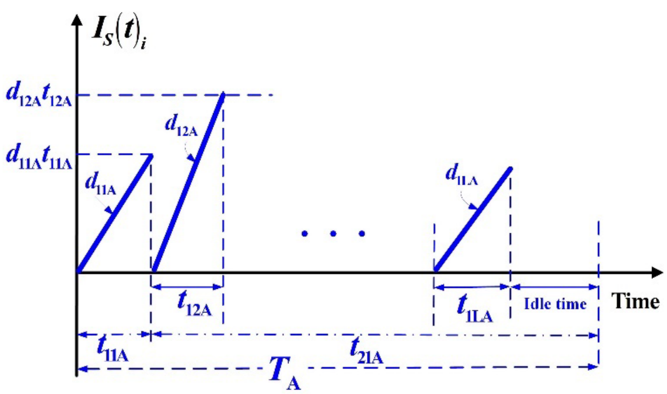

Figure 2. Inventory status of scrapped product *i* in the proposed system.

$$T_A = t_{1iA} + t_{2iA} \tag{4}$$
$$t_{1iA} = \frac{Q_i}{P_{1iA}} \tag{5}$$
$$H_i = (P_{1iA} - d_{1iA})t_{1iA} \tag{6}$$
$$t_{2iA} = T_A - t_{1iA} \tag{7}$$
$$d_{1iA}t_{1iA} = x_i P_{1iA} t_{1iA} = x_i Q_i \tag{8}$$
$$Q_i = \frac{\lambda_i T_A}{[1-E[x_i]]}. \tag{9}$$

When uptime ends, fixed size *n* installments of the completed lot of product *i* are delivered to retailer at t_{niA}. Inventory status of product *i* in delivery time t_{2iA} is exhibited in Figure 3 and total inventories during t_{2iA} are calculated as follows (Chiu et al., 2016):

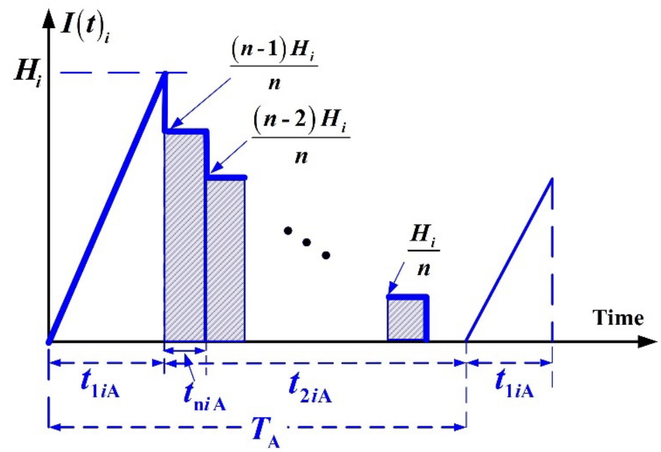

Figure 3. Inventory status of product *i* in delivery time t_{2iA} in the proposed multi-item producer-retailer integrated system.

(1) For $n = 1$, total inventories = 0.
(2) For $n = 2$, total inventories during t_{2iA} are as follows:

$$\left(\frac{H_i}{2} \times \frac{t_{2iA}}{2}\right) = \left(\frac{1}{2^2}\right) H_i t_{2iA} \tag{10}$$

(3) For $n = 3$, total inventories during t_{2iA} become

$$\left(\frac{2H_i}{3} \times \frac{t_{2iA}}{3} + \frac{H_i}{3} \times \frac{t_{2iA}}{3}\right) = \left(\frac{2+1}{3^2}\right) H_i t_{2iA} \tag{11}$$

Therefore, the following general term stands for total inventories of product *i* during t_{2iA}:

$$\left(\frac{1}{n^2}\right)\left[\frac{n(n-1)}{2}\right] H_i t_{2iA} = \left(\frac{n-1}{2n}\right) H_i t_{2iA} \tag{12}$$

At the retailer side, total stocks of product *i* can be computed as follows (details please see Appendix A):

$$\frac{1}{2}\left[\frac{H_i t_{2iA}}{n} + T_A(H_i - \lambda_i t_{2iA})\right] \tag{13}$$

3. System cost analysis

Contributors to the system cost $TC(T_A, n)$ comprise the following:

(1) Total setup, variable production, and disposal costs for *L* products.

$$\sum_{i=1}^{L}[K_{iA} + C_{iA}Q_i + C_{Si}(x_iQ_i)] = \sum_{i=1}^{L}[(1+\alpha_{2i})K_i + (1+\alpha_{3i})C_iQ_i + C_{Si}(x_iQ_i)] \tag{14}$$

(2) Total holding costs during uptime and shipping time.

$$\sum_{i=1}^{L}\left\{h_i\left[\frac{H_{1i}+d_{1iA}t_{1iA}}{2}(t_{1iA})+\left(\frac{n-1}{2n}\right)H_i(t_{2iA})\right]\right\} \tag{15}$$

(3) Total fixed and variable delivery costs.

$$\sum_{i=1}^{L}[nK_{1i}+C_{Ti}Q_i(1-x_i)] \tag{16}$$

(4) Total holding costs at retailer side.

$$\sum_{i=1}^{L}\frac{h_{2i}}{2}\left[\frac{H_{1i}t_{2iA}}{n}+T_A(H_i-\lambda_i t_{2iA})\right] \tag{17}$$

Therefore, $TC(T_A, n)$ is as follows:

$$TC(T_A,n)=\sum_{i=1}^{L}\left\{\begin{array}{l}(1+\alpha_{2i})K_i+(1+\alpha_{3i})C_iQ_i+C_{Si}(x_iQ_i)\\+h_i\left[\frac{H_{1i}+d_{1iA}t_{1iA}}{2}(t_{1iA})+\left(\frac{n-1}{2n}\right)H_i(t_{2iA})\right]\\+nK_{1i}+C_{Ti}Q_i(1-x_i)+\frac{h_{2i}}{2}\left[\frac{H_{1i}t_{2iA}}{n}+T_A(H_i-\lambda_i t_{2iA})\right]\end{array}\right\} \tag{18}$$

Using $E[x_i]$ to cope with randomness of x_i and replace Equations (4) to (9) in Equation (18), plus further computations, $E[TCU(T_A, n)]$ can be obtained:

$$E[TCU(T_A,n)]=\frac{E[TC(T_A,n)]}{E[T_A]}$$

$$=\sum_{i=1}^{L}\left\{\begin{array}{l}\frac{(1+\alpha_{2i})K_i}{T_A}+\frac{(1+\alpha_{3i})C_i\lambda_i}{1-E[x_i]}+C_{Si}\frac{\lambda_iE[x_i]}{1-E[x_i]}+C_{Ti}\lambda_i+\frac{nK_{1i}}{T_A}\\+\frac{(h_{2i}-h_i)T_A\lambda_i}{2n}\left[1-\frac{\lambda_i}{[1-E[x_i]](1+\alpha_{1i})P_{1i}}\right]\\+\frac{h_iT_A\lambda_i}{2}\left[1+\frac{E[x_i]\lambda_i}{[1-E[x_i]]^2(1+\alpha_{1i})P_{1i}}\right]+\frac{h_{2i}T_A\lambda_i^2}{2(1+\alpha_{1i})P_{1i}[1-E[x_i]]}\end{array}\right\} \tag{19}$$

or

$$E[TCU(T_A,n)]=\sum_{i=1}^{L}\left\{\begin{array}{l}\frac{(1+\alpha_{2i})K_i}{T_A}+\frac{(1+\alpha_{3i})C_i\lambda_i}{1-E[x_i]}+C_{Si}\frac{\lambda_iE[x_i]}{1-E[x_i]}+C_{Ti}\lambda_i+\frac{nK_{1i}}{T_A}\\+\frac{(h_{2i}-h_i)T_A\lambda_i(1-E_{1i})}{2n}+\frac{h_iT_A\lambda_i}{2}\left[1+\frac{E[x_i]E_{1i}}{[1-E[x_i]]}\right]+\frac{h_{2i}T_A\lambda_iE_{1i}}{2}\end{array}\right\} \tag{20}$$

where $E_{1i}=\frac{\lambda_i}{[1-E[x_i]](1+\alpha_{1i})P_{1i}}$.

4. Decision on rotation cycle time and frequency of delivery

The convexity of $E[TCU(T_A, n)]$ is first proved by Hessian matrix equations (Rardin, 1998), applying derivatives to Equation (20), we gain the following:

$$\frac{\partial^2E[TCU(T_A,n)]}{\partial T_A^2}=\sum_{i=1}^{L}\left\{\frac{2K_i(1+\alpha_{2i})}{T_A^3}+\frac{2K_{1i}n}{T_A^3}\right\} \tag{21}$$

$$\frac{\partial^2E[TCU(T_A,n)]}{\partial n^2}=\sum_{i=1}^{L}\left[\frac{(h_{2i}-h_i)T_A\lambda_i(1-E_{1i})}{n^3}\right] \tag{22}$$

$$\frac{\partial^2E[TCU(T_A,n)]}{\partial T_A\partial n}=\sum_{i=1}^{L}\left\{-\frac{K_{1i}}{T_A^2}-\frac{(h_{2i}-h_i)\lambda_i(1-E_{1i})}{2n^2}\right\} \tag{23}$$

Replace Equations (21) to (23) in Hessian matrix equations plus extra calculations, we found the following:

$$[T_A\quad n]\cdot\begin{pmatrix}\frac{\partial^2E[TCU(T_A,n)]}{\partial T_A^2} & \frac{\partial^2E[TCU(T_A,n)]}{\partial T_A\partial n}\\ \frac{\partial^2E[TCU(T_A,n)]}{\partial T_A\partial n} & \frac{\partial^2E[TCU(T_A,n)]}{\partial n^2}\end{pmatrix}\cdot\begin{bmatrix}T_A\\ n\end{bmatrix}=2\sum_{i=1}^{L}\left(\frac{(1+\alpha_{2i})K_i}{T_A}\right) \tag{24}$$

Because T_A, $(1+\alpha_{2i})$, and K_i are all positive, so Equation (24) is positive and $E[TCU(T_A, n)]$ is strictly convex for all n and T_A values other than zero. After proving convexity of $E[TCU(T_A, n)]$, next we concurrently locating optimal values of T_A^* and n^*, by setting the following first derivatives of $E[TCU(T_A, n)]$ regarding T_A and n equal to zero and solving the linear system:

$$\frac{\partial E[TCU(T_A,n)]}{\partial T_A}=\sum_{i=1}^{L}\left\{\begin{array}{l}\frac{-(1+\alpha_{2i})K_i}{T_A^2}-\frac{nK_{1i}}{T_A^2}+\frac{(h_{2i}-h_i)\lambda_i(1-E_{1i})}{2n}\\+\frac{h_i\lambda_i}{2}\left[1+\frac{E[x_i]E_{1i}}{[1-E[x_i]]}\right]+\frac{h_{2i}\lambda_iE_{1i}}{2}\end{array}\right\}=0 \tag{25}$$

$$\frac{\partial E[TCU(T_A,n)]}{\partial n}=\sum_{i=1}^{L}\left[\frac{K_{1i}}{T_A}-\frac{(h_{2i}-h_i)\lambda_iT_A(1-E_{1i})}{2n^2}\right]=0 \tag{26}$$

Accordingly, the following T_A^* and n^* are derived with extra derivation efforts:

$$T_A^*=\sqrt{\frac{2\sum_{i=1}^{L}[(1+\alpha_{2i})K_i+nK_{1i}]}{\sum_{i=1}^{L}\left\{\frac{(h_{2i}-h_i)\lambda_i(1-E_{1i})}{n}+h_i\lambda_i\left[1+\frac{E[x_i]E_{1i}}{[1-E[x_i]]}\right]+h_{2i}\lambda_iE_{1i}\right\}}} \tag{27}$$

$$n^*=\sqrt{\frac{\left(\sum_{i=1}^{L}[(1+\alpha_{2i})K_i]\right)\cdot\sum_{i=1}^{L}[\lambda_i(h_{2i}-h_i)(1-E_{1i})]}{\sum_{i=1}^{L}\{K_{1i}\}\cdot\sum_{i=1}^{L}\lambda_i\left\{h_i\left(1+\left[\frac{E[x_i]E_{1i}}{(1-E[x_i])}\right]\right)+h_{2i}E_{1i}\right\}}}. \tag{28}$$

4.1. Prerequisite capacity condition for the multiproduct fabrication planning

A prerequisite condition (Eq. (29)) for the multiproduct fabrication planning is a guarantee that production equipment has adequate capacity to make L products (Nahmias, 2009). In addition, another assumption (Eq. (30)) must also be true to avoid the unwanted stock-out situation for each end product *i* in the batch production planning.

$$\sum_{i=1}^{L}\left\{\frac{\lambda_i}{[1-E[x_i]]P_{1iA}}\right\} < 1 \tag{29}$$

$$P_{1iA} - d_{1iA} - \lambda_i > 0 \tag{30}$$

where $E[x_i]$ represents the expected scrap rate of product i (see the notation in subsection 2.1.)

4.2. The potential impact of the multiproduct fabrication setup times

The potential impact of the total setup times on the optimal rotation cycle length is as follows. In general, if the sum of setup times is small and can be fitted in the system's idle time, then the T_A^* (Eq. (27)) remains valid. Otherwise, one should compute the following T_{min} (as shown in Eq. (31)) and choose the actual operating cycle length T_A from max(T_A^*, T_{min}) to ensure that it can contain setup times (Nahmias, 2009).

$$T_A > \frac{\sum_{i=1}^{L}(S_i)}{1-\sum_{i=1}^{L}\left[\frac{\lambda_i}{[1-E[x_i]]P_{1iA}}\right]} = T_{min} \tag{31}$$

5. Numerical illustration

Assuming that five end items are to be produced in a multi-item producer-retailer integrated system featuring overtime and random scrap, and the relating variables used in this system are displayed in Table 1. Firstly, using Equations (27), (28), and (20), one gets $T_A^* = 0.5817$, $n^* = 3$, and $E[TCU(T_A^*, n^*)]$ = $2,758,443. The detailed investigative results on the impacts of differences in $\overline{\alpha_1}$ on distinctive system variables are exhibited in Table 2.

Examining the random scrap factor of the proposed multi-item producer-retailer integrated system, from Table 2, we find out that sum of quality cost is $208,655 (or about 7.56% of $E[TCU(T_A^*, n^*)]$); besides, the outcomes of examination on the impacts of variations in mean scrap rate *x* on the system performances, especially on $E[TCU(T_A^*, n^*)]$ and its associated cost factors are shown in Figure 4. It indicates that $E[TCU(T_A^*, n^*)]$ raises significantly, as average scrap rate *x* goes up; and major contributor to the cost increase is the quality cost, it boosts up drastically, as average *x* rises. It also confirms that at average $x = 0.15$, $E[TCU(T_A^*, n^*)]$ = $2,758,443.

Figure 5 depicts the investigative results on joint impacts of deviations in number of shipments per cycle *n* and rotation cycle time T_A on $E[TCU(T_A, n)]$. It exposes that $E[TCU(T_A, n)]$ notably increases, as both T_A and *n* depart from optimal points (i.e., $T_A^* = 0.5817$ and $n^* = 3$).

The influence of differences in the ratio of mean overtime unit cost over average ratio of $\overline{C_A}/\overline{C}$ on different fabrication cost for each item are studied, and Figure 6 exhibits its outcomes. It shows that as $\overline{C_A}/\overline{C}$ ratio increases, each item's different fabrication cost raises greatly.

Figure 7 illustrates exploratory results on joint influence of variations in mean scrap rate and mean overtime added proportion of production rate α_1 on $E[TCU(T_A^*, n^*)]$. It shows that $E[TCU(T_A^*, n^*)]$ radically raises, as both $\overline{x}$ and $\overline{\alpha_1}$ increase.

Figure 8 depicts analytical effects of variations in ratio of mean overtime expedited production rate over mean standard rate $\overline{P_{1A}}/\overline{P_1}$ along with different $\overline{x}$ on $E[TCU(T_A^*, n^*)]$. It shows that $E[TCU(T_A^*, n^*)]$ significantly increases, as $\overline{P_{1A}}/\overline{P_1}$ ratio raises; and as $\overline{x}$ goes up, $E[TCU(T_A^*, n^*)]$ rises.

Table 1. The relating variables used in the numerical illustration.

Item #	x_i	λ_i	P_{1i}	α_{1i}	P_{1iA}	C_i	α_{3i}	C_{iA}	K_i	α_{2i}	C_{Si}	h_i	K_{1i}	C_{Ti}	h_{2i}	K_{iA}
1	5%	3000	58000	0.30	75400	80	0.15	92	10000	0.06	20	10	2300	0.1	50	10600
2	10%	3200	59000	0.40	82600	90	0.20	108	11000	0.08	25	15	2400	0.2	55	11880
3	15%	3400	60000	0.50	90000	100	0.25	125	12000	0.10	30	20	2500	0.3	60	13200
4	20%	3600	61000	0.60	97600	110	0.30	143	13000	0.12	35	25	2600	0.4	65	14560
5	25%	3800	62000	0.70	105400	120	0.35	162	14000	0.14	40	30	2700	0.5	70	15960

Table 2. The impacts of differences in $\overline{\alpha_1}$ on distinctive system variables

$\overline{\alpha_1}$	n^*	T_A^*	$E[TCU(T_A^*, n^*)]$ [A]	% increase	Total variable production cost [B]	% [B]/[A]	% increase	Total quality cost [C]	% [C]/[A]	Sum of delivery cost	Sum of utiliza -tion	% decline	$\overline{\alpha_2}$	$\overline{\alpha_3}$
0.00	3	0.5566	$2,283,398	-	$1,720,000	75.33%	-	$209,047	9.16%	$72,673	0.3070	-	0.00	0.00
0.10	3	0.5622	$2,378,103	4.15%	$1,813,901	76.28%	5.46%	$208,941	8.79%	$72,007	0.2791	-9.09%	0.02	0.05
0.20	3	0.5674	$2,473,000	8.30%	$1,907,802	77.15%	10.92%	$208,852	8.45%	$71,393	0.2559	-16.67%	0.04	0.10
0.30	3	0.5723	$2,568,043	12.47%	$2,001,703	77.95%	16.38%	$208,777	8.13%	$70,820	0.2362	-23.08%	0.06	0.15
0.40	3	0.5771	$2,663,199	16.63%	$2,095,604	78.69%	21.84%	$208,712	7.84%	$70,281	0.2193	-28.57%	0.08	0.20
0.50	3	0.5817	$2,758,443	20.80%	$2,189,506	79.37%	27.30%	$208,655	7.56%	$69,771	0.2047	-33.33%	0.10	0.25
0.60	3	0.5861	$2,853,758	24.98%	$2,283,407	80.01%	32.76%	$208,605	7.31%	$69,286	0.1919	-37.50%	0.12	0.30
0.70	3	0.5904	$2,949,128	29.16%	$2,377,308	80.61%	38.22%	$208,561	7.07%	$68,821	0.1806	-41.18%	0.14	0.35
0.80	3	0.5945	$3,044,544	33.33%	$2,471,209	81.17%	43.67%	$208,522	6.85%	$68,374	0.1706	-44.44%	0.16	0.40
0.90	3	0.5986	$3,139,996	37.51%	$2,565,110	81.69%	49.13%	$208,486	6.64%	$67,944	0.1616	-47.37%	0.18	0.45
1.00	3	0.6026	$3,235,478	41.70%	$2,659,011	82.18%	54.59%	$208,455	6.44%	$67,528	0.1535	-50.00%	0.20	0.50
1.10	3	0.6065	$3,330,985	45.88%	$2,752,912	82.65%	60.05%	$208,426	6.26%	$67,125	0.1462	-52.38%	0.22	0.55
1.20	3	0.6104	$3,426,510	50.06%	$2,846,813	83.08%	65.51%	$208,399	6.08%	$66,735	0.1396	-54.55%	0.24	0.60
1.30	3	0.6142	$3,522,052	54.25%	$2,940,714	83.49%	70.97%	$208,375	5.92%	$66,355	0.1335	-56.52%	0.26	0.65
1.40	3	0.6179	$3,617,606	58.43%	$3,034,616	83.88%	76.43%	$208,353	5.76%	$65,985	0.1279	-58.33%	0.28	0.70
1.50	3	0.6216	$3,713,171	62.62%	$3,128,517	84.25%	81.89%	$208,332	5.61%	$65,624	0.1228	-60.00%	0.30	0.75
1.60	3	0.6253	$3,808,744	66.80%	$3,222,418	84.61%	87.35%	$208,313	5.47%	$65,273	0.1181	-61.54%	0.32	0.80
1.70	3	0.6289	$3,904,323	70.99%	$3,316,319	84.94%	92.81%	$208,296	5.34%	$64,929	0.1137	-62.96%	0.34	0.85
1.80	3	0.6324	$3,999,907	75.17%	$3,410,220	85.26%	98.27%	$208,280	5.21%	$64,593	0.1097	-64.29%	0.36	0.90
1.90	3	0.6360	$4,095,494	79.36%	$3,504,121	85.56%	103.73%	$208,264	5.09%	$64,265	0.1059	-65.52%	0.38	0.95
2.00	4	0.7053	$4,191,061	83.54%	$3,598,022	85.85%	109.19%	$208,301	4.97%	$76,192	0.1023	-66.67%	0.40	1.00

On the contrary, Figure 9 shows exploratory outcome on the effect of deviations in $\overline{P_{1A}}/\overline{P_1}$ ratio on individual machine utilization for each item. It discloses that individual utilization radically declines, as $\overline{P_{1A}}/\overline{P_1}$ ratio increases.

Figure 10 presents the joint influences of changes in n and average overtime unit cost connecting factor $\overline{\alpha_3}$ on $E[TCU(T_A, n)]$. It indicates that $E[TCU(T_A, n)]$ increases tremendously, as $\overline{\alpha_3}$ raises; and $E[TCU(T_A, n)]$ goes up, as n moves away from n^*.

Lastly, further research outcomes on the combined effects of changes in $\overline{x}$ and mean overtime added proportion of production rate $\overline{\alpha_1}$ on T_A^* are depicted in Figure 11. It shows that T_A^* declines slightly, as $\overline{x}$ increases; and as $\overline{\alpha_1}$ raises, T_A^* goes up notably. It is also noted that T_A^* climbs a step up, as $\overline{\alpha_1}$ raises to the range of [0.70 to 1.10], which is due to n^* changes from 3 to 4.

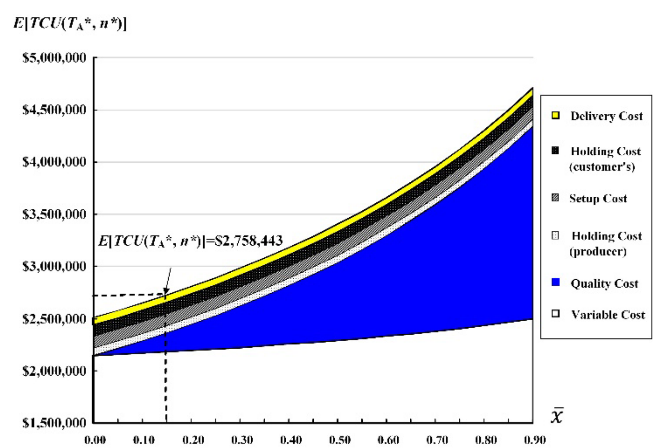

Figure 4. Effects of differences in $\overline{x}$ on $E[TCU(T_A^*, n^*)]$ and its associated cost factors.

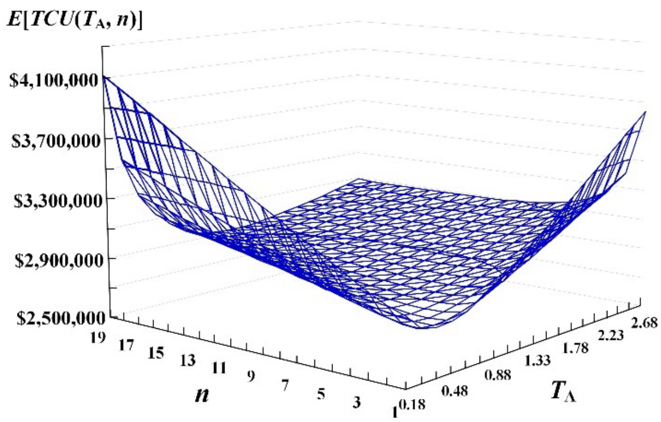

Figure 5. Joint impacts of deviations in number of deliveries n and cycle time T_A on $E[TCU(T_A, n)]$.

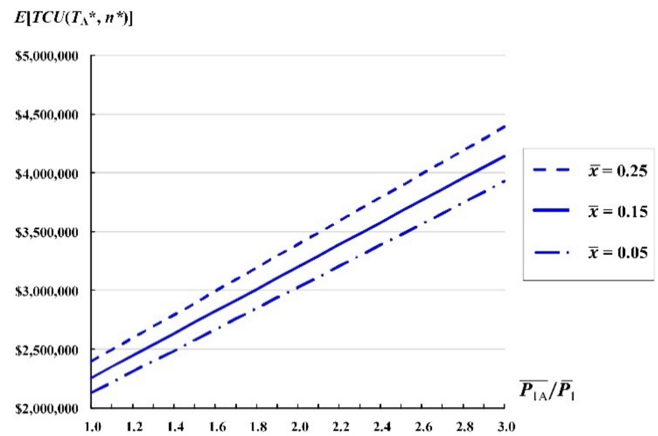

Figure 8. Effects of variations in $\overline{P_{1A}}/\overline{P_1}$ ratio along with different $\bar{x}$ on $E[TCU(T_A^*, n^*)]$.

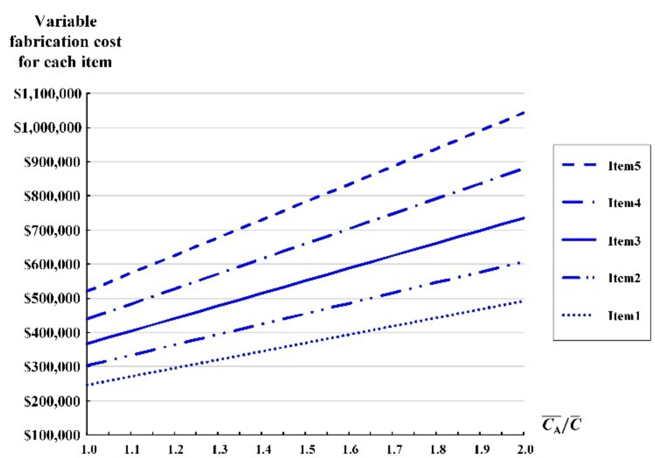

Figure 6. Impact of variations in $\overline{C_A}/\overline{C}$ on fabrication cost for each item.

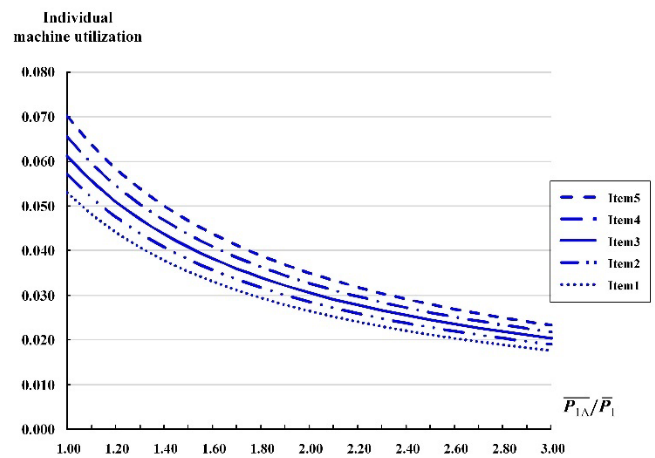

Figure 9. Effect of deviations in $\overline{P_{1A}}/\overline{P_1}$ ratio on individual machine utilization for each item.

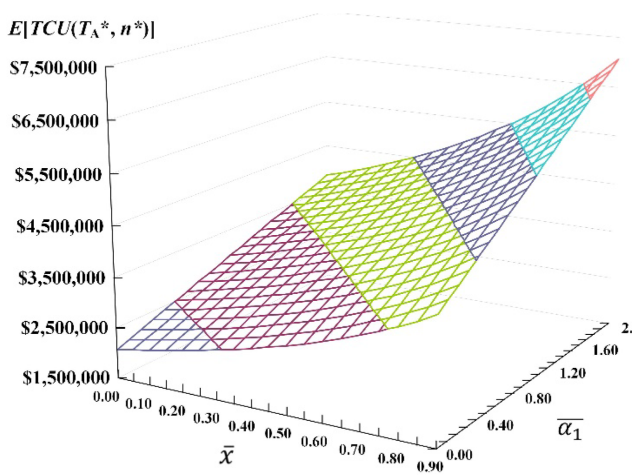

Figure 7. Joint effects of variations in $\bar{x}$ and $\overline{\alpha_1}$ on $E[TCU(T_A^*, n^*)]$.

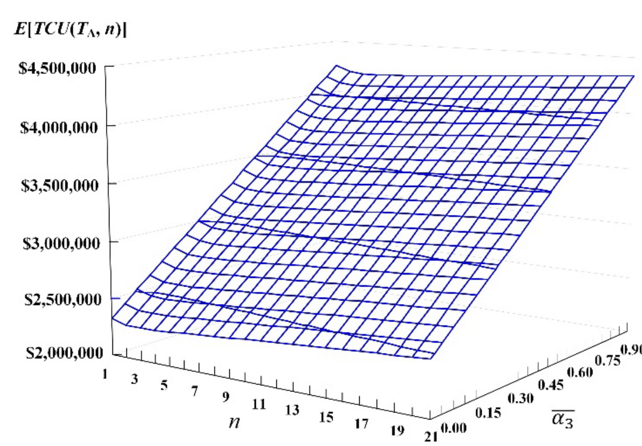

Figure 10. Joint impacts of changes in n and average cost connecting factor $\overline{\alpha_3}$ on $E[TCU(T_A, n)]$.

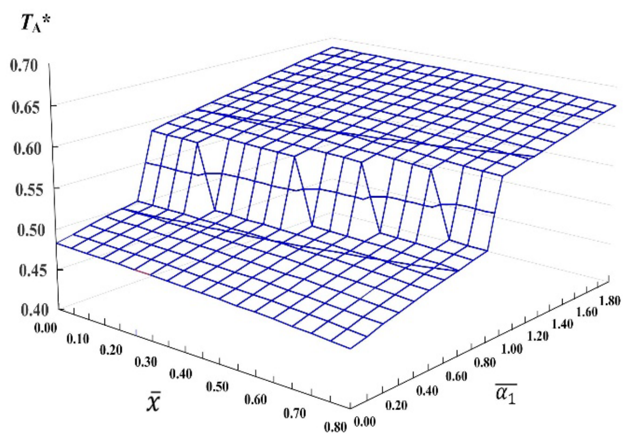

Figure 11. Combined impacts of differences in $\bar{x}$ and $\overline{\alpha_1}$ on T_A*.

6. Conclusions

The present study solves the rotation cycle length and delivery decision for a multiproduct producer-retailer integrated system featuring overtime and random scrap. Mathematical modeling and the Hessian matrix equations were applied to tackle the problem and determine the most economic stock refilling and shipping polices. Diverse important system information was revealed that can support managerial decision makings, which includes individual and combined influences of variations in particular system factor(s) (such as overtime related parameters and random scrap rate) on the specific system performance (refer to Figures 4 to 11). Without this profound study, essential system information will remain be concealed. Further investigation on issues of repairing defect items will be an interesting direction for future study.

Appendix A

Detailed calculation of total inventories at retailer side is given as follows:

First, the inventory status of product *i* at retailer side in the propose system is shown in Figure A-1. and the following equations can be clearly seen based on system description and assumption:

$$t_{niA} = \frac{t_{2iA}}{n} \quad \text{(A-1)}$$

$$D_i = \frac{H_i}{n} \quad \text{(A-2)}$$

$$I_i = D_i - \lambda_i t_{niA} \quad \text{(A-3)}$$

Accordingly, the inventories of product *i* at retailer side can be obtained from Figure A-1 as follows (Chiu et al., 2016):

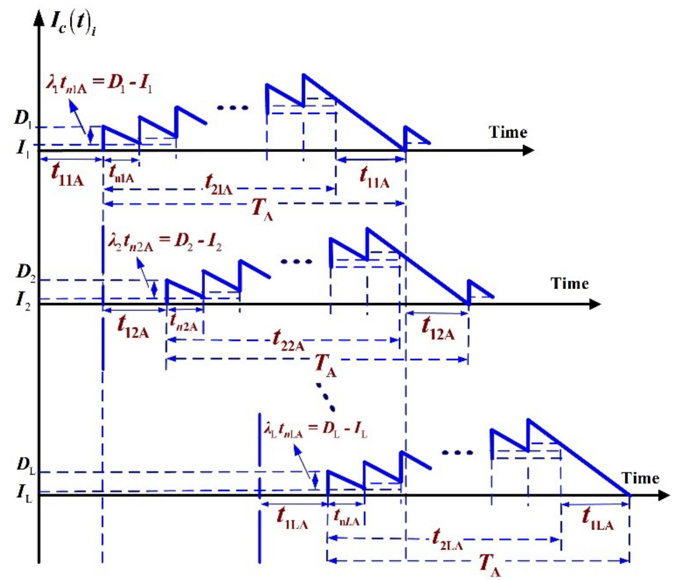

Figure A -1. Inventory status of product *i* at retailer side in the propose multi-item producer-retailer integrated system.

Accordingly, the inventories of product *i* at retailer side can be obtained from Figure A-1 as follows (Chiu et al., 2016):

$$\left[\frac{(D_i+I_i)t_{niA}}{2}\right]+\left[\frac{(D_i+I_i)+\left[(D_i+I_i)-\lambda_i t_{niA}\right](t_{niA})}{2}\right]$$
$$+\left[\frac{(D_i+2I_i)+\left[(D_i+2I_i)-\lambda_i t_{niA}\right](t_{niA})}{2}\right]+\cdots$$
$$+\left[\frac{\left[D_i+(n-1)I_i\right]+\left[\left[D_i+(n-1)I_i\right]-\lambda_i t_{niA}\right](t_{niA})}{2}\right]$$
$$+\frac{(nI_i)t_{1iA}}{2} \quad \text{(A-4)}$$

Substitute Eqs. (A-1) to (A-3) in Eq. (A-4) and with additional computations we obtain the following:

$$n\left(\frac{H_i}{n}-\frac{\lambda_i}{2}t_{niA}\right)t_{niA}+\frac{n(n-1)}{2}\left(\frac{H_i}{n}-\lambda_i t_{niA}\right)t_{niA}$$
$$+\frac{n}{2}\left(\frac{H_i}{n}-\lambda_i t_{niA}\right)(t_{1iA})$$
$$=\frac{H_i t_{2iA}}{n}-\frac{\lambda_i t_{2iA}^2}{2n}+\frac{H_i(n-1)t_{2iA}}{2n}-\frac{(n-1)\lambda_i t_{2iA}^2}{2n}$$
$$+\frac{H_i t_{1iA}}{2}-\frac{\lambda_i t_{2iA}}{2}(t_{1iA})$$
$$=\frac{1}{2}\left[\frac{H_i t_{2iA}}{n}+T_A\left(H_i-\lambda_i t_{2iA}\right)\right] \quad \text{(A-5)}$$

Conflict of interest

The authors do not have any type of conflict of interest to declare.

Financing

This research project was supported by the Ministry of Science & Technology of Taiwan under funding number MOST 106-2410-H-324-003 – A (special thanks from authors))).

References

Afshar-Nadjafi, B., Pourbakhsh, H., Mirhabibi, M., Khodaei, H., Ghodami, B., Sadighi, F., & Azizi, S. (2019). Economic production quantity model with backorders and items with imperfect/perfect quality options. *Journal of Applied Research and Technology, 17*(4), 250-257.
https://doi.org/10.22201/icat.16656423.2019.17.4.794

Barata, F. A. (2021). Performance measurement of supply chains and distribution industry using balanced scorecard and fuzzy analysis network process. *Decision Science Letters, 10*(3), 401-414.
https://doi.org/10.5267/j.dsl.2021.1.004

Bhagat, A., Sethi, R., & Garg, D. (2021). Controlled Arrival Machine Repair Problem with Working Vacation and Reattempts. *International Journal of Mathematical, Engineering and Management Sciences, 6*(1), 279-295.
https://doi.org/10.33889/IJMEMS.2021.6.1.018

Campbell, G. M. (2017). Time-staged overtime staffing for services with updated forecasts and availabilities. *Decision Sciences, 48*(4), 691-722.
https://doi.org/10.1111/deci.12237

Chiu, S. W., Hsieh, Y.T., Chiu, Y.S. P., & Hwang, M. H. (2016). A delayed differentiation multi-product FPR model with scrap and a multi-delivery policy – II: Using two-machine production scheme. International Journal for Engineering Modelling, 29(1-4): 53-68.

Chiu, Y. S.P., Chiu, V., Wang, Y., & Hwang, M. H. (2020). A postponement model for multi-item replenishment decision considering overtime, commonality, and quality reassurance. *International Journal of Industrial Engineering Computations, 11*(4), 509-524.
https://doi.org/10.5267/j.ijiec.2020.6.001

Chiu, Y. S.P., Lin, H. D., & Wu, H. Y. (2020). Note on Cost minimization for a multi-product fabrication-distribution problem with commonality, postponement and quality assurance. *Journal of Applied Research and Technology, 18*(1), 21-26.
https://doi.org/10.22201/icat.24486736e.2020.18.1.929

Chiu, S.W., Chiu, V., Hwang, M.H., & Chiu, Y.S.P. (2021). A delayed differentiation multiproduct model with the outsourcing of common parts, overtime strategy for end products, and quality reassurance. *International Journal of Industrial Engineering Computations, 12*(2), 143-158.
https://doi.org/10.5267/j.ijiec.2021.1.001

Chiu, Y., Chiu, T., Pai, F.Y., & Wu, H.Y. (2021). A producer-retailer incorporated multi-item EPQ problem with delayed differentiation, the expedited rate for common parts, multi-delivery and scrap. *International Journal of Industrial Engineering Computations, 12*(4), 427-440.
https://doi.org/10.5267/j.ijiec.2021.5.001

Daryanto, Y., & Christata, B. R. (2021). Optimal order quantity considering carbon emission costs, defective items, and partial backorder. *Uncertain Supply Chain Management, 9*(2), 307-316.
https://doi.org/10.5267/j.uscm.2021.3.002

Dixon, P. S., & Silver, E.A. (1981). A heuristic solution procedure for the multi-item, single-level, limited capacity, lot-sizing problem. *Journal of Operations Management, 2* (1), 23-39.
https://doi.org/10.1016/0272-6963(81)90033-4

El-Gohary, A., Tadj, L., & Al-Rasheedi, A. F. (2009). Using optimal control to adjust the production rate of a deteriorating inventory system. *Journal of Taibah University for Science, 2*(1), 69-77.
https://doi.org/10.1016/S1658-3655(12)60009-0

Eroglu, A., & Ozdemir, G. (2007). An economic order quantity model with defective items and shortages. *International Journal of Production Economics, 106*(2), 544-549.
https://doi.org/10.1016/j.ijpe.2006.06.015

Farmand, N., Zarei, H., & Rasti-Barzoki, M. (2021). Two meta-heuristic algorithms for optimizing a multi-objective supply chain scheduling problem in an identical parallel machines environment. *International Journal of Industrial Engineering Computations, 12*(3): 249-272.
https://doi.org/10.5267/j.ijiec.2021.3.002

Hariga, M., & Ben-Daya, M. (1998). Note: the economic manufacturing lot-sizing problem with imperfect production processes: bounds and optimal solutions. *Naval Research Logistics, 45*(4), 423-433.
https://doi.org/10.1002/(SICI)1520-6750(199806)45:4<423::AID-NAV8>3.0.CO;2-7

Ivanov, V., Dehtiarov, I., Pavlenko, I., Kosov, I., & Kosov, M. (2019). Technology for complex parts machining in multiproduct manufacturing. *Management and Production Engineering Review, 10*(2), 25-36.
https://doi.org/10.24425/mper.2019.129566

Kumar, M., Garg, D., & Agarwal, A. (2019). An analysis of inventory attributes in leagile supply chain: Cause and effect analysis. *International Journal of Mathematical, Engineering and Management Sciences, 4*(4), 870-881.
https://doi.org/10.33889/IJMEMS.2019.4.4-069

Lesmono, D., Limansyah, T., & Loedy, N. (2020). A joint return policy for a multi-item perishable inventory model with deterministic demands, return and all-units discount. *International Journal of Mathematical, Engineering and Management Sciences, 5*(3), 416-431.
https://doi.org/10.33889/IJMEMS.2020.5.3.035

Lin, H. D., Chen, Y. R., Chiu, V., & Chiu, Y. S.P. (2019). A decision model for a quality-assured EPQ-based intra-supply chain system considering overtime option. *Journal of Applied Engineering Science 17*(3), 362-371.
https://doi.org/10.5937/jaes17-19827

López-Ruíz, S., & Carmona-Benítez, R. B. (2019). The design of a liquefied natural gas (LNG) distribution network of a company operating in Mexico. *Journal of Applied Research and Technology,17*(4): 213-249.
https://doi.org/10.22201/icat.16656423.2019.17.4.818

Makino, H., & Tominaga, M. (1995). Estimation of production rate in flexible assembly systems. *CIRP annals, 44*(1), 7-10.
https://doi.org/10.1016/S0007-8506(07)62263-X

Mokao, R. (2020). Multi criteria decision making model for producing multiple products at the same time. Management and Production. *Engineering Review, 11*(3), 92-105.
https://doi.org/10.24425/mper.2020.134936

Mitchell, J.C. (1988). Multi-item inventory systems with a service objective. Operations Research, *36*(5), 747-755.
https://doi.org/10.1287/opre.36.5.747

Moussawi-Haidar, L., Salameh, M., & Nasr, W. (2013). An instantaneous replenishment model under the effect of a sampling policy for defective items. *Applied Mathematical Modeling, 37*(3), 719-727.
https://doi.org/10.1016/j.apm.2012.02.030

Nahmias, S. (2009). Production & Operations Analysis, 6th ed., McGraw-Hill Inc., New York.

Nicolaisen, H. (2011). Changes in the regulation of overtime under different collective bargaining regimes: a comparison of Irish, Norwegian and Swedish banking. *European Journal of Industrial Relations*, *17*(1), 7-23.
https://doi.org/10.1177/0959680110392274

Ohmori, S., & Yoshimoto, K. (2021). Multi-product multi-vehicle inventory routing problem with vehicle compatibility and site dependency: A case study in the restaurant chain industry. *Uncertain Supply Chain Management, 9*(2), 351-362.
https://doi.org/10.5267/j.uscm.2021.2.007

Pourmohammadi, F., Teimoury, E., & Gholamian, M. R. (2020). A scenario-based stochastic programming approach for designing and planning wheat supply chain (A case study). *Decision Science Letters, 9*(4): 537-546.
https://doi.org/10.5267/j.dsl.2020.8.004

Rardin, R.L. (1998). Optimization in Operations Research, Prentice-Hall, New Jersey.

Raza, S. A., & Govindaluri, S. M. (2019). Pricing strategies in a dual-channel green supply chain with cannibalization and risk aversion. *Operations Research Perspectives, 6*, 100118.
https://doi.org/10.1016/j.orp.2019.100118

Sambasivan, M., & Schmidt, C.P. (2002). A heuristic procedure for solving multi-plant, multi-item, multi-period capacitated lot-sizing problems. *Asia-Pacific Journal of Operational Research, 19*(1), 87-105.

Sana, S. S. (2010). A production–inventory model in an imperfect production process. *European Journal of Operational Research, 200*(2), 451-464.
https://doi.org/10.1016/j.ejor.2009.01.041

Sancak, E., & Salmann, F.S. (2011). Multi-item dynamic lot-sizing with delayed transportation policy. *International Journal of Production Economics 131*(2), 595-603.
https://doi.org/10.1016/j.ijpe.2011.02.004

Terdpaopong, K., Puapradit, A., & Malisuwan, U. (2021). The effect of lean on the operational performance of medium-sized Thai manufacturing companies. *Uncertain Supply Chain Management, 9*(3), 647-658.
https://doi.org/10.5267/j.uscm.2021.5.005

Yamada, T., Nagano, M., & Miyata, H. (2021). Minimization of total tardiness in no-wait flowshop production systems with preventive maintenance. *International Journal of Industrial Engineering Computations*, *12*(4), 415-426.
https://doi.org/10.5267/j.ijiec.2021.5.002